Substituting (i), (ii) and (iii) into (iv) gives:

$$i_p = -g_m i_p(1+k)R_K + g_s[E_s - i_p(1+k)R_K] + \frac{1}{r_p}\left(B + -\frac{\int i_p dt}{C}\right) \qquad \text{(v)}$$

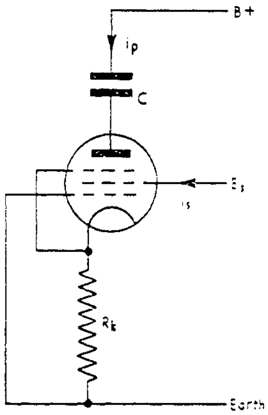

Fig. 3

Differentiating with respect to time and rearranging:

$$\frac{di_p}{dt} + \frac{i_p}{[(g_m+g_s)(1+k)R_K+1]r_pC} = 0$$

which is of the form:

$$\frac{di_p}{dt} + \frac{i_p}{RC} = 0 \qquad \text{(vi)}$$

where $R = [(g_m+g_s)(1+k)R_K+1]r_p$

Now
$$g_s = \frac{\delta i_p}{\delta e_s} = \frac{1}{k}\cdot\frac{\delta i_s}{\delta e_s}$$

where $\frac{\delta i_s}{\delta e_s} = R_s$, the screen resistance

$\simeq R_t$, the anode resistance of the valve when connected as a triode,

and $g_m r_p = \mu$

$$\therefore \quad R = \left[\left(\mu + \frac{r_p}{kR_s}\right)(1+k)R_K + r_p\right]$$

The solution of (vi) above gives

$$i_p = i_{p0}\epsilon^{-t/RC}$$

where i_{p0} = initial anode current of the valve.

An oscillation type magnetometer

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A simple method of measuring the saturation magnetization of thin plane samples of ferromagnetic materials is described. In this method the sample performs oscillations in a uniform magnetic field about an axis which lies in the plane of the sample and is at right angles to the magnetic field. The method was tested by measurements on sheet nickel and Supermalloy 0·11 mm thick and has been used with films of nickel down to 0·1 μ thick.

In the course of experiments on ferromagnetic resonance in thin films of nickel, the need became apparent for a good method of measuring the saturation magnetization of the thin films. The requirements were (*a*) that the measurements should be made on the actual specimen used in the resonance experiments (a circular disk of diameter 1·47 cm and thickness from 0·1 μ to 2·0 μ), (*b*) that, preferably, an homogeneous magnetic field should be used and (*c*) that the accuracy should be of the order of 2–5%.

The following method was found to satisfy these requirements and to provide valuable additional information. The disk, mounted in a holder, is suspended by a torsionless fibre in a uniform magnetic field, with the plane of the disk vertical. Then the equilibrium condition is that in which the plane of the disk is parallel to the magnetic field, and the disk will oscillate about this position when given a slight initial rotation. The equation of motion is then $K\ddot{\theta} + T\theta = 0$ neglecting damping, where K is the moment of inertia of the disk and its holder about the axis of rotation and T is the torque per unit angular displacement from the equilibrium position. Thus the period of oscillation $\tau = 2\pi\sqrt{(K/T)}$ and T depends on the magnetic moment of the disk.

To find T, consider the film at a small angle θ to the magnetic field and take axes as x, the axis of rotation, y normal to the film and z perpendicular to x and y. The following assumptions are now made:—

(1) that the magnetic field H is sufficient to produce magnetic saturation of the film;
(2) that the magnetization vector I per unit volume does not alter in magnitude but only in direction as θ varies; and
(3) that the direction of I is such as to make the total magnetic energy of the film a minimum.

There are two energy terms, that due to the interaction of the magnetic moment of the film with the external field, which is $-\boldsymbol{I}\cdot\boldsymbol{H}$ per unit volume, and the internal energy due to the demagnetizing field H_{int}, which is $-\frac{1}{2}\boldsymbol{I}\cdot\boldsymbol{H}_{int}$ per unit volume. When the specimen is an ellipsoid or can be approximated thereto, $\boldsymbol{H}_{int}$ can be written as $-NI$ where N is the demagnetizing factor along a principal axis of the ellipse.

Let $\boldsymbol{I}$ make an angle ϕ with the z axis, lying between z and the magnetic field. Then the external energy per unit volume is given by

$$E_{ext} = -IH\cos(\theta - \phi)$$

and the internal energy by

$$\begin{aligned} E_{int} &= \tfrac{1}{2}N_zI_z^2 + \tfrac{1}{2}N_yI_y^2 = \tfrac{1}{2}N_zI^2\cos^2\phi + \tfrac{1}{2}N_yI^2\sin^2\phi \\ &= \tfrac{1}{2}N_zI^2 + \tfrac{1}{2}(N_y - N_z)I^2\sin^2\phi \end{aligned}$$

where N_y and N_z are the demagnetizing factors in the y and z directions respectively. The value of ϕ for which the total energy $E = E_{ext} + E_{int}$ is a minimum is then given by

$$\frac{\partial E}{\partial \phi} = 0 = -IH\sin(\theta - \phi) + (N_y - N_z)I^2\sin\phi\cos\phi$$

or since both θ and ϕ are very small

$$-H(\theta - \phi) + (N_y - N_z)I\phi = 0$$

Therefore $$\phi = \theta\frac{H}{H + (N_y - N_z)I} \qquad (1)$$

The torque on the film of volume v is

$$T\theta = v\boldsymbol{I} \times \boldsymbol{H} = vIH\sin(\theta - \phi) \simeq vIH(\theta - \phi)$$

and from (1) $$= \frac{vIH}{\dfrac{H}{(N_y - N_z)I} + 1}\theta$$

Hence the periodic time of oscillation is given by

$$\tau^2 = \frac{4\pi^2K}{vIH}\left[\frac{H}{(N_y - N_z)I} + 1\right] \qquad (2)$$

Thus I may be found if $(N_y - N_z)$ is known since all the other quantities can be measured.

Alternatively equation (2) may be written

$$\frac{\tau^2 vI}{4\pi^2K} = \frac{1}{(N_y - N_z)I} + \frac{1}{H} \qquad (3)$$

and therefore if I is independent of H, there is a linear relation between τ^2 and $1/H$ from which both $(N_y - N_z)$ and I may be found.

The experimental arrangement used was very simple. The disk was mounted in a holder made of non-conducting material (Tufnol) to avoid eddy current damping, and suspended by a fine Nylon thread, so that the axis of rotation was a diameter of the disk. Another thread attached to the bottom of the holder prevented any sideways motion. The size and therefore the moment of inertia of the holder was chosen so that the period τ lay in the range 0·3 to 5 sec for the particular specimen and fields used. This made it possible to use hand and eye methods for measuring τ. To facilitate such measurements a small mirror made by evaporating a thin layer of aluminium or silver on to a thin piece of mica was stuck on to one face of the holder and used with a lamp and scale. The holder was made of two parts of equal length cut from a solid cylinder of Tufnol and made so that one half screwed tightly into the other, thus clamping the disk in the centre. Three different holders were used, of diameters 3·19, 2·20 and 2·80 cm and of lengths 0·63, 3·21 and 5·60 cm, and moments of inertia 4·26, 19·1 and 145 g cm^2 respectively.

Most of the measurements were made on the thin evaporated films of nickel, but to test the method, measurements were also made on disks of annealed nickel and Supermalloy sheet of 0·11 cm thickness and 1·47 cm diameter. It was found that a plot of τ^2 against $1/H$ gave a straight line within the accuracy of the experiment, except the points for nickel at a field strength less than 1 500 gauss which are referred to below. Typical values were: $\tau = 0{\cdot}477$ and 0·536 sec at $H = 5\,200$ and 3 530 gauss respectively for nickel and $\tau = 0{\cdot}431$ and 0·595 sec at $H = 4\,100$ and 1 710 gauss respectively for Supermalloy, using the holder of moment of inertia $K = 145$ g cm^2. From these measurements the two quantities $vI/4\pi^2K$ and $(N_y - N_z)I$ may be determined either graphically or by the method of least squares. The values of $4\pi I$ obtained from the first quantity for nickel and Supermalloy were 6·0 and $7{\cdot}9 \times 10^3$ respectively.

To find I from the second quantity, it is necessary to know $(N_y - N_z)$ and in this case it is assumed that the sheets are essentially strain free, so that the demagnetizing factors are entirely due to shape. A good approximation for the demagnetization factor in the plane of a thin disk is given by $N_z = \epsilon = \pi^2/r$ where r is the ratio of diameter to thickness of the disk. Then $N_y = 4\pi - 2\epsilon$ since $N_x + N_y + N_z = 4\pi$ for static fields and hence $(N_y - N_z) = (4\pi - 3\epsilon)$. In this case r is 134 and $\epsilon = 0{\cdot}0737$ and the values of $4\pi I$ for nickel and Supermalloy found by this method were 6·2 and $7{\cdot}9 \times 10^3$ respectively.

Both these methods give values which agree, within experimental error, with the room temperature values of $6{\cdot}2 \times 10^3$ for nickel and $7{\cdot}9 \times 10^3$ for Supermalloy taken from the literature.

The following points on the use of this method may be noted:—

(*a*) It has been assumed above that I is independent of H and therefore H must be large enough to ensure saturation of the sample. For nickel, points taken at $H <$ about 1 500 gauss were found to lie off the straight line and were discarded.

(*b*) The effect of damping on the period has been neglected in the above treatment, but it can easily be measured and allowed for in the usual way. In fact, this correction was found to be negligible.

(*c*) As described above, equation (3) has been used to find the value of I in two ways, but it may also be used to measure both I and $(N_y - N_z)$. This is of particular importance for use with ferro-magnetic resonance experiments, where it is the quantity $(N_y - N_z)I$ which occurs in the expression for the resonance condition.[1] In some cases the specimen may be anisotropic due either to crystalline anisotropy or to the effect of strain. This will affect the period of oscillation, but in the simple case where the anisotropy has uniaxial symmetry with the axis normal to the plane of the specimen, it may be included in the demagnetization coefficients and measured in this way.[2]

(*d*) The accuracy is mainly dependent on the accuracy of measurement of the magnetic field, which in these experiments was about ½–1%. In the particular case of the measurements on nickel this gives an error of about 3% in the value of I measured by the first method, to which must be added error in v, the volume of the sample, and K the moment of inertia of the system. For the second method the error is about 2% and this is independent of K and v. The accuracy achieved in any particular measurement depends on the range of magnetic field strengths used and therefore cannot be given as a general statement.

(*e*) In the present series of experiments, films of nickel ranging in thickness from 0·1 μ to 2 μ and Supermalloy and nickel sheets of thickness 110 μ have been measured. The method could certainly be extended outside this range particularly for thicker specimens.

The results of measurements by this method on thin nickel films together with the results of the ferromagnetic resonance experiments will be published in the near future.

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The measurement and time integration of small voltages and currents

The instrument designed by I. A. D. Lewis and A. C. Clark for the measurement of the current and charge output of the University of Liverpool cyclotron* has proved very useful for a similar purpose in connexion with the Yale University linear electron accelerator. However, difficulty was experienced with the circuit, which was traced to an inherent erratic instability of the mutual conductance of the Mullard type EF50 valves. A large quantity of these valves has been tested and all show the same effect: when subjected to even very slight mechanical shock the mutual conductance of the valve will shift to another value which it will retain until shaken again. These valves are used as d.c. amplifiers and cause a fluctuation of the calibration of this circuit.

We have replaced V_5, V_6, and V_7 in the circuit with R.C.A. type 6SH7 valves, which, while microphonic, will return to their initial mutual conductance value after shock. This change has necessitated the change of three resistors. The new values are: R_{27}, 47 kΩ; R_{28}, 11 kΩ; and R_{29}, 68 kΩ. The 90 V screen potential for the 6SH7's is obtained through a dropping resistor from the 225 V supply.

To use the circuit for electrons instead of positive particles, the leads from the cathodes of V_1 and V_3 to the grids of V_5 and V_6 have been interchanged with completely satisfactory results. We have also found it convenient to use a telephone minor switch with gold-plated contacts in place of the bank of range-changing relays used by Lewis and Clark. This is controlled by a telephone dial. All insulation in the input circuit has been replaced with Kel-F (similar to Teflon) with the result that there has been no trouble due to low insulation resistance. A point which was not clear in the original paper is that the potential of Point A in the circuit must be re-set (with a vacuum-tube voltmeter) to within 0·1 V of the correct potential every hour or so in order to maintain a linear relation of counts versus charge. The correct potential must be found empirically for the particular instrument. For our circuit the potential was − 2·0 V.

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A current integrator

It is usual to use a Faraday cage to measure the number of charged particles in the beam from a cyclotron or similar equipment. The charge collected is allowed to leak through a resistor to a current integrating device. Currents may be integrated using a capacitor and thyratron valve. The current charges the capacitor to a fixed potential at which the thyratron fires, discharging the capacitor and recording a count. Because the striking potential of thyratron valves is not constant the stability of these circuits is poor; further, the minimum current which can be integrated is not less than 10^{-10} A. There have been several attempts to overcome these difficulties.[1,2,3] The circuit described here has several advantages: (1) simple, requiring no special components; (2) low leakage current; (3) high stability.

The valve V_1 is connected so that the cathode potential is