This disadvantage is common to both the balanced and unbalanced bridges, but patient selection from a sufficiently large supply of diodes should reduce it to a minimum.

The dynamic power range of this correlator is relatively small. If the input signal strength exceeds about ± 0.1 volt, the values of A and γ in the crystal characteristic change and distortion is introduced. Conversely, too small a signal is lost in the noise generated in the diodes. Also, since this correlator is a passive device, the output is of small magnitude (a few microamperes) and must be measured with a sensitive galvanometer or amplified with a differential dc amplifier.

In the analysis of the unbalanced bridge operation it was shown that the output must be averaged over an interval which is long with respect to the input voltage variations, if the nonproduct terms in the output are to appear as a bias. Thus the unbalanced bridge is a rapid-response multiplier, not an instantaneous multiplier. This limitation is no disadvantage, when bridge is included with prior delay and post averaging in a correlator.

A correlator including this unbalanced bridge multiplier is usable over a very wide frequency range; the limits are those of the input cables themselves and their terminations. It is simple in construction and operation, and the constituent parts are not given to drift over long periods of time.

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The Calibration of Amplitude Modulation Meters with a Heterodyne Signal*

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Summary—**Errors which may occur in the calibration of peak or average reading amplitude modulation meters with a heterodyne signal are investigated in detail. Relations between the heterodyne-amplitude ratio *M* and the true modulation factor *m* of a sinusoidally modulated wave are derived. Using these results, heterodyne calibration of modulation analyzers to read *m* exactly can readily be ·ied out.**

INTRODUCTION

AMPLITUDE-MODULATION ANALYZERS for intermodulation measurements at audio frequency[1,2] or for the determination of modulation depth in radio transmission[3] are commonly calibrated with a heterodyne signal produced by summing two signals of different amplitudes and frequencies whose frequency difference is small compared to their frequencies. The reason for the use of such a calibration signal in place of a truly sinusoidally modulated signal is that the latter is difficult to produce with known modulation factor *ab initio*, whereas the former is easily produced with an accurately known ratio between the amplitudes of the beating signals. Since it is apparently not generally recognized that appreciable errors can arise when this calibration procedure is employed, this paper discusses how such errors occur and how they can be eliminated.

PRINCIPLES OF MODULATION MEASUREMENT

Most modulation analyzers read directly in terms of modulation factor and to do so perform the following operations on an input signal. First, the signal passes through a high-pass filter which removes any low-frequency modulating component which may be present and leaves the higher-frequency modulated carrier intact. In order to allow a final indication in terms of modulation factor alone, the average amplitude of the modulated carrier is set to a given level determined during initial calibration of the instrument. After necessary amplification, the modulated carrier is then demodulated with either a half- or full-wave linear detector. The rectified output of the detector then passes through a low-pass filter which eliminates unmodulated signal components at the carrier frequency and higher. The signal at the output of this filter then consists of the low-frequency modulating component of the modulated wave together with a dc component which is next eliminated by a series capacitor.

The resulting output signal is the low-frequency modulation alone. By comparing its amplitude with that of the carrier, the modulation factor may be directly obtained. In intermodulation testing, the modulating signal is usually rectified with a full-wave linear

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[1] J. K. Hilliard, "Distortion tests by the intermodulation method," Proc. I.R.E., vol. 29, pp. 614–620; December, 1941.

[2] J. M. van Beuren, "Simplified intermodulation measurements," *Audio Engineering*, vol. 34, p. 24; November, 1950.

[3] F. E. Terman, "Radio Engineers' Handbook," First Edition, p. 987, McGraw-Hill, New York, N. Y.; 1943.

rectifier and its rms amplitude read on a dc meter, which responds to the average value of the rectified signal. On the other hand, modulation analyzers for rf signals commonly provide means to measure the amount of positive and negative peak modulation separately. They therefore use a half-wave detector and peak reading dc output meter which can be switched to read either positive or negative peaks of the modulating signal.[3]

Analysis of Heterodyne Calibration

In this section, we shall show that errors can arise from calibration with a heterodyne signal whether an output meter measuring the peak or average of the rectified-modulating signal is employed. The errors are appreciably smaller in the latter case but arise in both cases from the difference in waveshape between a true sinusoidally-modulated wave and a heterodyne wave.

The equation for a sinusoidally modulated wave may be written

$$\begin{aligned} e_0(t) &= A\left[1 + m \cos \omega_m t\right] \cos \omega_c t \\ &= A\left[\cos \omega_c t + \frac{m}{2} \cos (\omega_c + \omega_m) t \right. \\ &\quad \left. + \frac{m}{2} \cos (\omega_c - \omega_m) t\right], \end{aligned} \tag{1}$$

where A is a constant and m is the modulation factor, which ranges between zero and unity. The modulation analyzer separates out the modulating signal

$$e_m(t) = Am \cos \omega_m t, \tag{2}$$

and the output reading is proportional to its peak amplitude Am (the same for either positive or negative peaks) or its average amplitude, $2Am/\pi$ for full-wave rectification, Am/π for half-wave rectification. Assuming that the modulation factor m of the above signal is known, we shall first discuss how the modulation analyzer can in theory be calibrated to read m directly, then compare the procedure necessary when a heterodyne signal instead of a sinusoidally modulated signal is used for such calibration.

First, the signal of (1) is applied to the analyzer, whose gain is then adjusted so that the modulation meter reading corresponds to the known m. With the same gain, a reading R proportional to the average value of the modulated carrier (1) after rectification is then obtained either with a separate meter or by bypassing the first detector and reading the result on the modulation meter. Since this average value is simply A, independent of m, the analyzer will now yield the correct value of m for any other modulated signal like (1) but with different A and m, so long as the gain of the analyzer is first adjusted to give the same reading R for the average value of the rectified-modulated carrier. We see that this calibration procedure effectively removes the factor $2A/\pi$ or A/π occurring in the average value of the rectified-modulating signal by normalizing the average amplitude of the modulated carrier to a given value. A similar procedure may be used for a peak-reading output meter.

A pure, sinusoidally modulated signal of known modulation factor is seldom available for calibration purposes. On the other hand, however, a heterodyne signal $e_h(t)$ may be readily formed by adding the two signals $e_1(t) = A \cos \omega_c t$ and $e_2(t) = AM \cos (\omega_c + \omega_m)t$ to yield

$$\begin{aligned} e_h(t) &= A\left[\cos \omega_c t + \frac{M}{2} \cos (\omega_c + \omega_m) t \right. \\ &\quad \left. + \frac{M}{2} \cos (\omega_c + \omega_m) t\right] \\ &= A\left[(1 - M) \cos \omega_c t \right. \\ &\quad \left. + 2M \cos\left(\frac{\omega_m t}{2}\right) \cos\left(\frac{2\omega_c + \omega_m}{2}\right) t\right], \end{aligned} \tag{3}$$

where M is the ratio of the smaller signal e_2 to the larger, e_1.[4] M may range between zero and unity, and the incorrect assumption is often made that it is equal to the modulation factor m for a true sinusoidally-modulated wave. Comparison of the second form of (1) and the first form of (3) shows, however, that although the signals are very similar, the modulated wave has upper and lower sidebands, whereas the heterodyne wave has only an upper (or lower) sideband. We shall now derive relations between M and m for positive or negative peak or for average-reading modulation meters which will allow correct calibration to be achieved with a heterodyne-calibrating signal for any value of M.

In order to obtain relations between M and m, it is necessary to investigate in detail what happens to the heterodyne signal as it passes through the various stages in the modulation analyzer. Fig. 1, on the next page, gives a comparison of the forms of a sinusoidally modulated wave with $m = 1$ in (a) and a heterodyne wave with $M = 1$ in (b). The difference between these waveshapes is very small for m and $M \ll 1$, and is a maximum when these quantities equal unity. After passing through a linear half- or full-wave detector and a low-pass filter, the waveform in (c) is obtained. The filter must, of course, have a sufficiently wide pass band that the sharp minima in (c) be passed with negligible distortion, yielding an accurate reproduction of the positive envelope of the heterodyne wave. Its equation for arbitrary M, which is readily obtained by a vector addition of $e_1(t)$ and $e_2(t)$, is

$$e_e(t) = \left[A^2 + M^2A^2 + 2MA^2 \cos \omega_m t\right]^{1/2}. \tag{4}$$

If we let $\omega_m t = 2\theta$ and $k^2 = 4M/(1+M)^2$, (4) may be rewritten as

$$e_e(\theta) = A(1 + M)\left[1 - k^2 \sin^2 \theta\right]^{1/2}. \tag{5}$$

The average value of this envelope is

[4] M.I.T. Electrical Engineering Staff, "Applied Electronics," p. 699, John Wiley and Sons, Inc., New York, N. Y.; 1943.

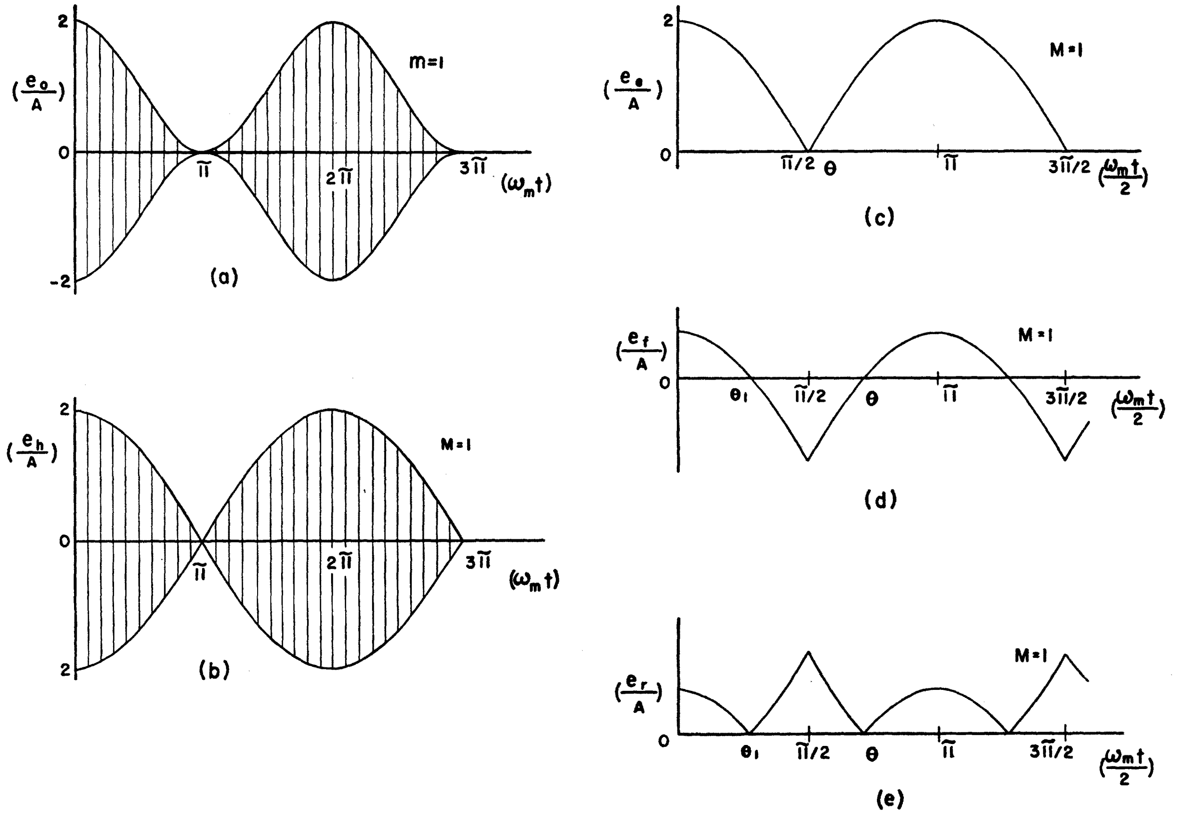

Fig. 1—Waveshapes in modulation analyzer. (a) Sinusoidally modulated input signal. (b) Heterodyne input signal. (c) Envelope of rectified heterodyne signal. (d) Envelope of rectified heterodyne signal with dc component removed. (e) Full-wave rectified waveshape of signal in (d).

$$\bar{e}_e = \frac{2A(1+M)}{\pi}\int_0^{\pi/2}[1 - k^2\sin^2\theta]^{1/2}d\theta = \frac{2A(1+M)}{\pi}E, \tag{6}$$

where E is the complete elliptic integral of the second kind. Note that as M approaches zero, $2E/\pi$ approaches unity and $\bar{e}_e$ approaches A, the value obtained for a sinusoidally modulated wave. For $M>0$, however, $2E/\pi$ is a function of M and is less than unity.

When the waveform (5) is passed through a condenser, the dc component is eliminated and the equation for the resulting signal $e_f(\theta)$ is

$$e_f(\theta) = A(1+M)[\sqrt{1 - k^2\sin^2\theta} - 2E/\pi]. \tag{7}$$

The waveshape of $e_f(\theta)$ for $M=1$ is shown in Fig. 1(d). It is now necessary to determine the value of θ for which $e_f(\theta)=0$; this value, θ_1, is readily found to be

$$\theta_1 = \sin^{-1}\left\{\frac{1}{k}[1 - (2E/\pi)^2]^{1/2}\right\}. \tag{8}$$

θ_1 approaches $\pi/4$ as M tends to zero and increases slightly above this value for nonzero M. The positive and negative peaks of the waveform of (7) occur at $\theta=0$ and at $\theta=\pi/2$ and are

$$e_f(0) = A(1+M)(1 - 2E/\pi) \tag{9}$$

$$e_f(\pi/2) = A(1+M)(\sqrt{1-k^2} - 2E/\pi). \tag{10}$$

If the signal of (7) is full-wave rectified, the waveshape between $\theta=0$ and $\theta=\pi/2$ is given by

$$\begin{aligned} e_r(\theta) &= e_f(\theta) \quad 0 \leqq \theta \leqq \theta_1 \\ &= -e_f(\theta) \quad \theta_1 \leqq \theta \leqq \pi/2, \end{aligned} \tag{11}$$

and is shown in Fig. 1(e) for $M=1$. A dc meter will respond to the average value of this rather peculiar wave. This average value is

$$\bar{e}_r = \frac{2A(1+M)}{\pi}\int_0^{\pi/2} e_r(\theta)d\theta = \frac{4A(1+M)}{\pi}\int_0^{\theta_1} e_f(\theta)d\theta. \tag{12}$$

The last equation follows because the area under the curve from $\theta=0$ to θ_1 is necessarily equal to that from $\theta=\theta_1$ to $\pi/2$. The result of the above integration is

$$\bar{e}_r = [4A(1+M)/\pi][E(\theta_1, k) - 2\theta_1 E/\pi], \tag{13}$$

where $E(\theta_1, k)$ is the incomplete elliptic integral of the second kind.

We are now finally in a position to obtain the relation between the true modulation factor m and the heterodyne signal ratio M. First, we shall consider an analyzer which measures the average value of $e_r(\theta)$. Part of the calibration procedure, as discussed above, involves adjusting the gain of the analyzer so that the rectified-modulated carrier gives a certain set reading on a dc meter. For a sinusoidally modulated wave, this reading is independent of m, but for a heterodyne wave it depends on M, as shown by (6). This dependence can introduce error of its own into the calibration with a heterodyne signal unless it is properly taken into account. Because of this potential error and because of differences between the waveshape of a sinusoidally modulated wave and of a heterodyne wave, m is not equal to M. We define $\bar{m}$ in the present average-reading case as the true modulation factor for a sinusoidally modulated wave corresponding to a given M for a heterodyne wave. The calibration procedure using a heterodyne signal therefore consists of adjusting the analyzer gain so that the heterodyne signal gives a modulation reading of $\bar{m}$ (not M) and then reading a quantity R proportional to the average value of the rectified-modulated carrier $\bar{e}_e$. After such calibration, the analyzer will measure the true modulation factor m for a sinusoidally modulated signal when the carrier amplitude is initially set to R.

For the above type of analyzer which measures a quantity proportional to $\bar{e}_r$, the actual modulation-meter reading, after setting the rectified modulated-carrier-level to the proper value will be $\bar{m}$, by definition, where $\bar{m}$ is given by $\bar{e}_r/\bar{e}_e$. We therefore obtain

$$\bar{m} = (2E/\pi)^{-1}[2E(\theta_1, k) - 4\theta_1 E/\pi]. \qquad (14)$$

The value of M corresponding to $\bar{m}$ in (14) is, of course, implicit in E, $E(\theta_1, k)$, k, and θ_1.

For an analyzer which measures positive and negative peaks of the modulating voltage $e_f(\theta)$, the actual modulation-meter readings, again after setting the proper modulated-carrier level, are m^+ for positive peaks, m^- for negative, where these quantities are given by $e_f(0)/\bar{e}_e$ and $-e_f(\pi/2)/\bar{e}_e$, respectively. The results are

$$m^+ = (2E/\pi)^{-1} - 1 \qquad \text{(positive peak reading)} \quad (15)$$

$$m^- = 1 - \sqrt{1 - k^2}\,(2E/\pi)^{-1} \quad \text{(negative peak reading)}. \quad (16)$$

Eqs. (14), (15), and (16) are the most important results of this analysis and allow the different types of modulation meters to be calibrated to read m correctly with a heterodyne wave. In Table I we tabulate the dependence of $\bar{m}$, m^+, and m^- on M for $M = 0$ to 1. The percentage differences of these quantities from M are plotted versus M in Fig. 2. These calculations were carried out using the tables of elliptic functions given by Jahnke and Emde.[5] Fig. 2 indicates that less than one per cent error in calibration will occur when M is taken equal to m in the range $M < 0.04$ for m^+ and m^-, and $M < 0.17$ for $\bar{m}$. In addition, a negative peak reading meter can also be calibrated with less than one per cent error with M taken equal to m^- when $0.96 < M \leq 1$. On the other hand, if meters reading m^+ or $\bar{m}$ are calibrated near $M = 1$, the error in a modulation reading near $M = 1$ will be of the order of 30 or 40 per cent if M is erroneously assumed to equal m.

TABLE I

Dependence of $\bar{m}$, m^+, and m^- on Heterodyne Signal Ratio M

M	$\bar{m}$	m^+	m^-
0	0	0	0
0.007654	0.00765	0.007628	0.007680
0.03110	0.0311	0.03084	0.03133
0.07180	0.07160	0.07039	0.07302
0.1325	0.1316	0.1276	0.1362
0.2174	0.2136	0.2032	0.2266
0.3333	0.3183	0.2970	0.3515
0.4903	0.4467	0.4045	0.5196
0.7041	0.5796	0.5102	0.7378
1	0.6613	0.5708	1

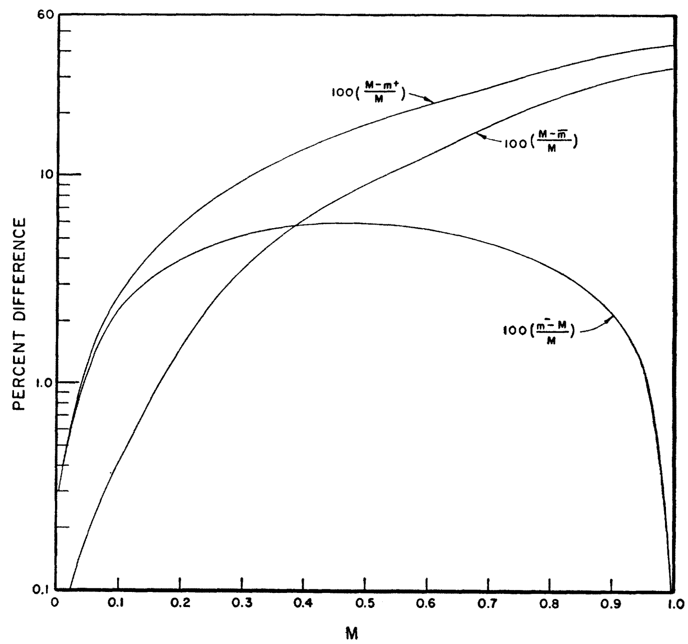

Fig. 2—Percentage difference between true modulation factors $\bar{m}$, m^+, and m^- and heterodyne amplitude ratio M as functions of M.

Conclusions

The calibration of amplitude-modulation analyzers with a heterodyne-calibrating signal has been analyzed in detail and potential errors implicit in such calibration have been investigated. It has been found that the error may be as large as 40 per cent if the heterodyne-amplitude ratio M is taken equal to the true modulation factor m of a sinusoidally modulated wave when M is near unity.

Essentially errorless calibration with a heterodyne signal can be carried out in either of two ways. First, if M and m are assumed equal but a value of M much less than unity is used, negligible error will result. A second method, which is exact for any value of M, requires only that the value of m obtained either from Table I or calculated from the formulae derived in this paper be used in the calibration procedure, instead of M.

[5] E. Jahnke and F. Emde, "Tables of Functions," Dover, N. Y.; 1943.