LETTER TO THE EDITOR

Physical realizability conditions

Pelzer[1]) has recently called in question a statement in earlier work [2]) of the present authors wherein it is mentioned that the usual conditions for physical realizability on $Q(\omega)$, the reduced polarization or network function, are too restricted.

The primary aim of the earlier work was to show how the Kronig-Kramers integral transform relations could be derived by Mellin transforms without recourse to the usual methods of complex integration which lead directly to the physical realizability conditions. It was stated that the treatment was formal only. The above conclusion that the conditions of physical realizability are too restrictive of the form of $Q(\omega)$ was intended to indicate that the real and imaginary parts of $Q(\omega)$ could be properly connected by the Kronig-Kramers relations even when some of these conditions are not satisfied. This conclusion was not connected with the question of physical realizability of the system described by $Q(\omega)$. It is unfortunate that because our phraseology was unsufficiently explicit here Pelzer was led to the conclusion that we were abjuring some or all of the usual conditions of physical realizability for a physically realizable system. Instead, we were directing attention from a formal viewpoint to a possible class of $Q(\omega)$'s which represent non-physically realizable systems, yet whose real and imaginary parts satisfy the Kronig-Kramers relations.

In later work carried out in the Spring of 1955, we came to the conclusion, in agreement with Pelzer, that our example of such behavior, the Fuoss-Kirkwood dielectric [3]), was inadequate and is not a member of the above class when it is required that both Kronig-Kramers relations hold. We did not then communicate this conclusion because it formed a small part of a much more extensive paper then being prepared for publication [4]).

Many cases can be found where one or the other but not both of the Kronig-Kramers relations hold for a non-physically realizable system. A simple example is given by $Q(p) = p$, where $p = \sigma + i\omega$ is a complex frequency variable. Here $Q(\omega) \equiv \lim_{\sigma \to 0} Q(p) = 0 + i\omega$ is non-analytic at infinity. As discussed in greater detail in Ref. 4, we have only been able to make both Kronig-Kramers relations hold for some non-physical (e.g. non-analytic) systems by using a mathematical artifice, and in general, we suggest that the above class is null. In Ref. 4, we also discuss the Fuoss-Kirkwood and Cole-Cole dielectrics in more detail and show, as does Pelzer, that the real and imaginary parts of the corresponding reduced polarizations can be made to correspond to physical systems satisfying the usual realizability conditions provided the real and imaginary parts of the $Q(\omega)$'s are written as even and odd functions of ω respectively by introducing absolute value signs and sign (ω) functions where necessary.

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