Tuning of a Rectangular Parallelopiped Cavity Resonator with a Circular Metallic Post

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In radar T-R tubes and in ferromagnetic resonance experiments, it is often desirable to tune a rectangular cavity resonator by means of a circular metallic post projecting into one wall. Here it is shown how a perturbation formula due to Slater may be employed to calculate the relation between resonant frequency and small displacements of such a metallic post; the calculation is carried out to first order for a rectangular cavity operating in the TE_{110}-mode and the result compared with a similar relation for a right circular cylinder whose length is varied by a plunger completely closing one end.

I. INTRODUCTION

The tuning of a right circular cylindrical cavity resonator is readily accomplished by means of a micrometer screw arranged to alter the length of the cavity.[1] The micrometer drives a cylindrical plunger which fits the cavity bore fairly closely. By incorporating several quarter-wave short-circuiting choke sections on the plunger, its rotation and axial motion produce little or no changes in the electrical termination at the end of the cavity.

In a rectangular cavity, it would also often be convenient to tune the cavity by altering one of its dimensions. Since an end of a rectangular cavity cannot be rotated as in a cylindrical cavity, the wall must be coupled to a micrometer screw through a coupling which eliminates the rotation of the micrometer. Such a coupling is complicated and may introduce backlash which would make calibration of the tuning uncertain. An alternative used in approximately rectangular cavities such as those of T-R tubes makes use of a small tuning screw in one wall. Such a screw can, of course, only produce a small change in resonant frequency.

In ferromagnetic resonance experiments, it is often desirable to use a rectangular cavity rather than a circular cylindrical one.[2,3] In such cases, it is desirable that the microwave magnetic field be perpendicular to an applied static magnetic field. Exact perpendicularity over the full end cannot be achieved for any mode of oscillation in a circular cylindrical cavity but can be produced easily in the TE_{110} mode of a rectangular cavity. Since changes in the external static magnetic field produce changes in the loaded Q of such a cavity, one or more of whose walls is made of ferromagnetic material, the cavity must be retuned to resonance after a change in magnetic field strength.

Over a limited frequency range, tuning in a rectangular cavity may be accomplished by means of a micrometer and plunger as in the circular cylindrical cavity. Provided the plunger is round and of less diameter than the smaller of the two dimensions of the wall into which it projects, there will be no need to eliminate the rotation of the micrometer and the plunger can be allowed to rotate as it progresses into the cavity. Under such conditions, calibration can be made both accurate and reproducible.

Although calibration of tuning of the above nature can always be carried out experimentally when necessary, it is desirable for the design of such tuning plungers to have an equation relating the change of resonant frequency to the motion of the plunger. The present work furnishes such an equation for a rectangular cavity operating in the TE_{110}-mode and indicates the method of procedure for obtaining similar relations for other modes. Usually it is desirable that the cavity continue to operate primarily in a given mode over the entire tuning range. Since the present method of tuning involves the projection of a metallic circular cylinder out from a rectangular wall, any finite amount of projection will require a complicated new mode structure to allow the boundary conditions to be satisfied. Therefore, in the present work, we restrict the analysis to very small positive or negative deviations of the plunger from its flush position. For such small deviations the added modes will be of negligible practical importance and may be neglected.

II. METHOD OF CALCULATION AND DISCUSSION OF RESULTS

The calculation of the tuning formula is carried out in the appendix using a theorem due to Slater[4] which states that an infinitesimal variation of the volume of a resonant cavity causes the following change in resonant frequency:

$$\frac{\Delta\omega_0}{\omega_0}=-\frac{\Delta\lambda_a}{\lambda_a}=-\frac{\Delta E_s}{2E_s}, \tag{1}$$

where E_s is the energy stored in the cavity and

$$\Delta E_s=\tfrac{1}{2}\int_{\Delta V}\{\mathbf{H}\cdot\mathbf{H}^*-\mathbf{E}\cdot\mathbf{E}^*\}dV; \tag{2}$$

[1] C. G. Montgomery, *Technique of Microwave Measurements* (McGraw-Hill Book Company, Inc., New York, 1947), p. 319 ff.
[2] J. R. Macdonald, PhD thesis, Oxford University, 1950 (unpublished).
[3] J. R. Macdonald, Proc. Phys. Soc. (London) **A64**, 968 (1951).
[4] J. C. Slater, Revs. Modern Phys. **18**, 441 (1946). Our Eq. (1) follows from Slater's equation III—89 rewritten in terms of frequency deviations.

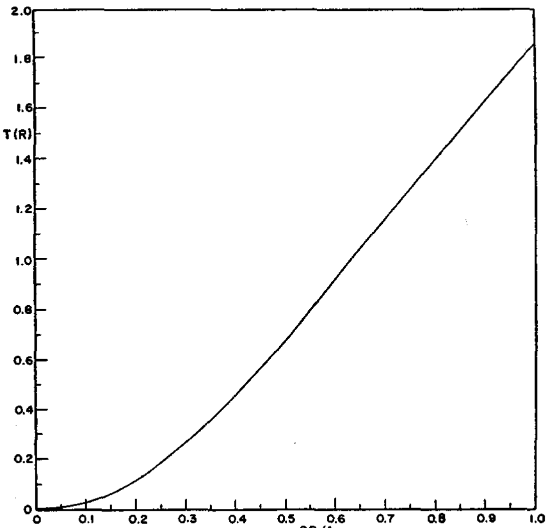

FIG. 1. Dependence of the tuning factor $T(R)$ on $2R/A$ for a rectangular cavity operating in the TE_{110}-mode.

the integration is over the infinitesimal change in cavity volume. In Eq. (1) λ_a is the free-space wavelength corresponding to the resonant frequency ω_0.

The result of the calculation for a rectangular resonator of dimensions A, B, C operating in the TE_{110}-mode is

$$\Delta\lambda_a=\left\{\frac{A\eta^2}{2\pi C}[1+(2/\eta)J_1(\eta)]\right\}(\lambda_a/\lambda_g)^3\Delta l, \qquad (3)$$

where Δl is the displacement of a metal plunger of radius R in the direction of increasing volume parallel to the B dimension, and λ_g is the cavity wavelength $2B$. The quantity η is $2K_1R$, where $K_1=\pi/A$.

It is now of interest to compare Eq. (3) with the corresponding equation which applies to a right circular cylindrical resonator whose tuning is accomplished by a micrometer screw and plunger which alters the length. By differentiating the fundamental relation

$$\lambda_a^{-2}=\lambda_g^{-2}+\lambda_c^{-2}, \qquad (4)$$

one immediately obtains

$$\Delta\lambda_a=(\lambda_a/\lambda_g)^3\Delta\lambda_g, \qquad (5)$$

where λ_c in (4) is the cut-off wavelength of the cavity. For the TE_{111}-mode of the cylindrical cavity, for which $\Delta l=\Delta\lambda_g/2$, (5) becomes

$$\Delta\lambda_a=2(\lambda_a/\lambda_g)^3\Delta l. \qquad (6)$$

Note that the value of λ_g in (6) is different from that in (3). We have chosen the cylindrical TE_{111}-mode for comparison because it is often used in paramagnetic and ferromagnetic resonance experiments since it yields the closest approximation to constant parallel microwave magnetic field lines over the cavity ends.

The principal difference between (3) and (6) lies in the term within curly brackets in (3) which is replaced by 2 in (6). In Fig. 1, we plot the term $T(R)=(\eta^2/2\pi)\times[1+(2/\eta)J_1(\eta)]$ in (3) which contains the R-dependence of the equation *versus* the factor $2R/A$. Note that when A is taken equal to C, the term A/C drops out of (3) and the factor in curly brackets will be a maximum for the maximum value of R. From Fig. 1 we see that even with the maximum possible value of unity for $2R/A$, the factor is only 1.856 instead of 2, as in the right circular cylindrical cavity. This result is not surprising, of course, since the circular plunger cannot cover the entire wall of the rectangular cavity as it does in the cylindrical cavity.

Equation (3) and Fig. 1 allow the tuning characteristics of a circular cylindrical tuning post or plunger to be used in designing such tuning for a rectangular cavity operating in the TE_{110}-mode. Such a cavity has been constructed with the above type of tuning and the formula (3) verified experimentally[2] over the range of λ_a from 1.23 to 1.27 cm. The calculations upon which the present work are based were carried out by the author in 1949.[2]

APPENDIX

Consider a rectangular resonator operating in the TE_{110}-mode and having dimensions A, B, and C in the x, y, and z directions. Tuning is to be accomplished by small displacements $\pm\Delta l$ in the $\pm y$-direction of a cylindrical plunger of diameter $2R(\leqslant A,C)$ whose face, of the same metal as the cavity walls, is normally flush with the wall at $y=0$. In applying Eq. (1), we need consider, to first order, only the field H_x since all other components are zero at $y=0$. When H_x at $y=0$ is expressed in terms of cylindrical coordinates r,θ centered on the plunger axis at $x=A/2$, $y=B/2$, we obtain[5]

$$(H_x)_{y=0}=(K_1K_2/K^2)\cos(K_1r\cos\theta), \qquad (7)$$

where $K_2=\pi/B$, and $K^2=(\pi/A)^2+(\pi/B)^2=2\pi/\lambda_a$. If now Eq. (7) is substituted into (2), the result simplified, and the y-integration carried out, one finds

$$\begin{aligned}\Delta E_s&=\frac{\Delta l}{4}\left(\frac{K_1K_2}{K^2}\right)^2\int_0^R\int_0^{2\pi} r\{1+\cos(2K_1r\cos\theta)\}d\theta dr\\&=\frac{\pi\Delta l}{2}\left(\frac{K_1K_2}{K^2}\right)^2\int_0^R r\{1+J_0(2K_1r)\}dr.\end{aligned} \qquad (8)$$

The integral in Eq. (8) may now be evaluated to yield

$$\Delta E_s=\frac{\pi R^2\Delta l}{4}\left(\frac{K_1K_2}{K^2}\right)^2\left\{1+\frac{1}{K_1R}J_1(2K_1R)\right\}. \qquad (9)$$

Finally, on substituting (9) and the well-known result $E_s=ABC(K_1/K)^2/8$ into (1), we obtain Eq. (3).

[5] See reference 1, p. 295.