One-dimensional current transport equations *

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Because the divergence of the total current (all conduction and displacement terms included) is zero, the space average of the current for one-dimensional flow is equal to the current itself, even in the presence of generation-recombination and trapping effects. This result is helpful because the average form of the current is frequently easier to use in static and time-varying problems than its unaveraged form. The space invariance of the current is explicitly illustrated for a typical one-dimensional transport situation with recombination.

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One of Maxwell's equations leads directly to the condition $\nabla \cdot \mathbf{I} = 0$, where I includes all conduction and displacement currents. Thus, this total current is space invariant everywhere, a particularly valuable condition in boundary-value problems. Here, a useful consequence of this result is outlined for one-dimensional current flow situations.

Consider a situation where the electric current density may be well approximated by its x component only. Assume that the current flowing over a path along the x axis involves a single species of positive charge carriers of concentration p, mobility μ_p, and valence number z_p, and a single species of negative carriers of concentration n, mobility μ_n, and valence number z_n. The specific current, or current density, may then be written

$$I = I_n + I_p + I_d\ , \tag{1}$$

where the conduction terms I_n and I_p are given in terms of the specific particle fluxes J_n and J_p by

$$I_n = -\, e z_n J_n \tag{2}$$

and

$$I_p = e z_p J_p , \tag{3}$$

where e is the proton charge. Further, the specific displacement current will be expressed here as

$$I_d = (\epsilon / 4\pi)\dot{E}, \tag{4}$$

where E is the electric field, ϵ is the bulk dielectric constant (taken to be space and frequency independent in the range of interest), and a superscript dot denotes partial differentiation with respect to time, $\partial/\partial t$.

When space charge is present, and especially near interfaces, I_n, I_p, and I_d will generally all be functions of x. But in the one-dimensional approximation without sources and sinks, I itself clearly cannot depend on x. Let partial differentiation with respect to x, $\partial/\partial x$, be denoted by a superscript prime. Then $I' = 0$ in this case, even though I_n', I_p', and I_d' need not be zero at any specific x value. Many space-charge calculations[1–4] have made use of this constancy of I by forming

$$\langle I \rangle \equiv L^{-1} \int_0^L I(x)\, dx = I, \tag{5}$$

where L is a part of the current path, say that between two plane parallel electrodes containing a material of interest between them. This approach is frequently particularly useful because integration removes the x dependencies of the individual components of I and allows this total current to be expressed in terms of the $(0, L)$ boundary values of such quantities as n and p.

To what degree does the above simplifying artifice apply when generation/recombination (G/R) is present in the material of interest? One might expect that G/R would, in general, introduce x-dependent sources and sinks, even in a one-dimensional approximation, making it perhaps not intuitively obvious (except to Maxwell) that Eq. (5) would continue to hold, either dynamically or statically. Nevertheless, Eq. (5) has usually been applied without discussion even in G/R cases.[1–4] Since the present author is unaware of previous consideration of this question, the following treatment shows the validity of (5) explicitly.

The usual somewhat idealized transport equation approximations for the specific particle fluxes in the one-dimensional approximation are given by

$$J_n = -\,(\mu_n n E + D_n n') \tag{6}$$

and

$$J_p = (\mu_p p E - D_p p'), \tag{7}$$

where D_n and D_p are diffusion coefficients, here taken position independent. As we shall see, however, explicit forms for J_n and J_p, such as these, are not needed for the present treatment. Now, Poisson's equation for a specific case of interest is[5]

$$E' = (4\pi e/\epsilon)(z_p p - z_n n + N_e), \tag{8}$$

where the term $N_e \equiv N_D^+ - N_A^-$ accounts for charged donors and acceptors, assumed always fulled ionized. Thus $\dot{N}_e = 0$.

Next, assume that contributions to the n and p concentrations arise from dissociation of intrinsic neutral centers of concentration $n_c(x)$. Since dissociation of these centers leads to intrinsic conductivity, full ionization will never be reached. It is the generation and recombination of these possibly mobile charged species, which are here assumed to be of the same kinds as those arising from impurity donors and acceptors, which provides the G/R terms we shall consider.

For concreteness, assume that the dissociation of a single neutral center leads to z_{pL} positive charges of valence number z_p and z_{nL} negative charges of valence number z_n, where the neutrality of the dissociating center requires that $z_{pL} z_p = z_{nL} z_n$. For definiteness, further assume that $\dot{n}_c$ is made up of an ordinary generation term and a "multimolecular" recombination term following from the law of mass action for the particles involved. Then,

$$\dot{n}_c = -k_1 n_c + k_2 (n^{z_{nL}})(p^{z_{pL}}), \tag{9}$$

where k_1 is a generation coefficient (temperature and possibly light dependent) and k_2 is a recombination coefficient. This specific form of $\dot{n}_c$ above is provided for illustration; it is not needed in proving Eq. (5).

Finally, the continuity equations for n and p may be written

$$\dot{p} = -z_{pL}\dot{n}_c - J_p' \tag{10}$$

and

$$\dot{n} = -z_{nL}\dot{n}_c - J_n' . \tag{11}$$

A sufficient but not necessary condition for Eq. (5) to hold is

$$I' = 0 \tag{12}$$

for all x in the range of interest, $0 \leq x \leq L$. Form I' from (1), $\dot{E}'$ from (8), and use these results together with Eqs. (2)–(4), (10), and (11). One then obtains

$$I' = -ez_n J_n' + ez_p J_p' + (\epsilon/4\pi)\dot{E}'$$
$$= e(z_n z_{nL} - z_p z_{pL})\dot{n}_c = 0, \tag{13}$$

Thus, Eq. (5) holds even in the presence of G/R (and/or trapping) and may be used to simplify current and impedance calculations in this case. Fundamentally, Eq. (5) and the underlying condition $I' = 0$ are associated with charge conservation; in both general G/R and trapping any charge created or destroyed by the process always sums to zero. Note further that since the specific idealized forms of J_n, J_p, and $\dot{n}_c$ given by Eqs. (6), (7), and (9) were not needed in the proof, it is quite unnecessary that D_n', D_p', μ_n', μ_p' all be zero. Thus for all times adequately described by the present equations (no retarded effects considered), the total current I is constant in space even in time-varying situations.

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