plotted against any possible variable. The plotting of the results versus slew rate is legitimate, but it must then be borne in mind that the relative level of the THD signal is 14 dB higher than that of the DIM signal, resulting in correspondingly earlier clipping in the amplitude domain. In our paper we plotted the results against the output voltage, which is the standardized way of presentation.

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COMMENTS ON "COMPUTER-AIDED LOUDSPEAKER SYSTEM DESIGN PART 1: SYNTHESIS USING OPTIMIZATION TECHNIQUES"[1]

This interesting, useful paper on computer-aided loudspeaker system design by G. J. Adams shows how one may synthesize desired loudspeaker characteristics by determining basic loudspeaker parameters using an optimization technique. This note is written to point out that another more powerful technique for such synthesis exists. The method to be described is applicable to the estimation of values of the vector of parameters $\boldsymbol{x} = (x_1, x_2, \cdots, x_k, x_{k+1}, \cdots, x_p)$ which occur in a complex function (impedance, admittance, transfer function, etc.) of ω ($\equiv 2\pi f$), say $H(\omega, \boldsymbol{x})$, representing a model, or theoretical representation, of a physical system. Here, as in Adams,[1] it is assumed that there are k free parameters and $(p - k)$ fixed ones. Suppose the free parameters of this model are to be determined by comparing it with another complex function $G(\omega)$. This function in Adams's case was a known specified high-pass filter function, but it may be actual data measured on an experimental system. In this case $G(\omega_i)$ denotes the real and imaginary parts of experimental measurements, at each ω_i, of the complex quantity corresponding in the physical system to $H(\omega_i, \boldsymbol{x})$.

Adams minimized the objective function $\epsilon(\boldsymbol{x}) = \Sigma W_i [|G(\omega_i)| - |H(\omega_i, \boldsymbol{x}|]^2$, a typical weighted least-square functional, by a direct search method. Although he suggests that, alternatively, a combination of both amplitude and phase responses could be minimized, he does not follow up this suggestion and uses only an amplitude comparison. Although in minimum-phase systems, such as that considered by Adams, no new information is afforded by phase knowledge if amplitude is known for all frequencies, even here actual data will have random errors in both amplitude and phase, and a simultaneous comparison of both to estimate the vector of values of $\boldsymbol{x}$ makes use of all available data, throws nothing away, and may thus be expected to yield statistically better defined parameter estimates. Further, in the non-minimum-phase situation, Kronig–Kramers (Bode) relations do not allow phase to be determined from amplitude or vice versa; so amplitude comparison is theoretically insufficient. Even in Adams's case of a comparison with exact "data" in a minimum-phase situation, it is very likely that parameter determination will be more sensitive (that is, fewer steps to achieve convergence to best estimates and possibly smaller statistical uncertainties of the resulting estimates) when both real and imaginary information concerning a complex function is simultaneously used in the objective function. Note that these quantities may be, for the complex function $H(\omega)$, any of the pairs $\{\text{mod}[H(\omega)], \arg[H(\omega)]\}$; $\{\text{Re}[H(\omega)], \text{Im}[H(\omega)]\}$; $\{\text{Re}[H^{-1}(\omega)], \text{Im}[H^{-1}(\omega)]\}$, etc. If $H(\omega)$ were an input impedance, for example, then $H^{-1}(\omega)$ would be the corresponding input admittance.

A method to carry out the above model-parameter estimation has been developed[2] and extensively investigated. It uses any pair (as above) $\{H_a(\omega_i), H_b(\omega_i)\}$, a composite vector which is itself a function of the vector $\{\omega\}$, and compares it by means of a weighted least-square objective function to determine the best estimates of the x_j and their standard deviations. Nonlinear least squares is used for the optimization procedure; thus the x_i may appear nonlinearly in $H(\omega, \boldsymbol{x})$ in any form or combination. Arbitrary weighing may be employed, and it is assumed that the ω_i are known with negligible uncertainty. The method is a slight generalization of ordinary nonlinear least squares and is described in detail elsewhere.[2] In some sense it is the inverse of multiple regression, where one dependent variable y depends on several independent variables $z_1, z_2, \cdots$. Here the quantities $H_a(\omega, \boldsymbol{x})$ and $H_b(\omega, \boldsymbol{x})$, whose dependences on $\boldsymbol{x}$ will generally be different, may be thought of as two dependent variables, both dependent upon the single independent variable ω. The actual procedure involves forming a composite dependent-variable vector made up of both H_a and H_b.

It is recommended that this approach be employed as an alternative to that described by Adams and, in general, for any data-fitting or model-parameter determination tasks involving paired observations such as the real and imaginary parts of an impedance versus frequency, temperature, potential, or other independent, causative variable accurately measured. In actual tests of a partly distributed electric circuit model involving up to eight free and initially unknown parameters and measured impedance-frequency data obtained from a real physical system expected to be well described by the model, all eight of the parameters were well determined with relative standard deviations usually appreciably less than 0.1. In favorable cases even more free parameters should be determinable.

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[1] G. J. Adams, *J. Audio Eng. Soc.*, vol. 26, pp. 826–837 (1978 Nov.).

[2] J. R. Macdonald and J. A. Garber, *J. Electrochem. Soc.*, vol. 124, pp. 1022–1030 (1977 July).