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THREE DIMENSIONAL PERSPECTIVE PLOTTING AND FITTING OF IMMITTANCE DATA

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Three dimensional perspective plotting of impedance or admittance data illustrates directly the interdependencies between the three variables: real part, imaginary part, and frequency. In addition, the three planar projections included show the associated three relationships between any two of these variables. We illustrate the usefulness and versatility of such plotting for synthetic data derived from given equivalent circuits and for data obtained from measurements on lumped-constant circuits and on ionic conductors. The utility of nonlinear complex least squares data fitting is demonstrated by a 3-D plot which compares both the original data and calculated "data" obtained from the fitting results.

Although such conventional plots as parallel capacitance and conductance and tan(δ) vs frequency, and/or impedance or admittance plane plots, are useful in illustrating some aspects of the small-signal electrical response of solids and liquids, they are two-dimensional representations of three-dimensional phenomena. Thus any single such plot cannot show the full response of the system. In this paper we illustrate some of the virtues of perspective views of three-dimensional representations of such three-dimensional phenomena. Unlike ordinary two-dimensional plots, perspective plots can show details of the full response at a glance.

Suppose that the measured quantity (an impedance, admittance or even a complex dielectric constant) is denoted by I(f), where f is the frequency. Then one may write $I(f) = |I(f)| \measuredangle I(f) = \mathrm{Re}[I(f)] + i\mathrm{Im}[I(f)]$. We have elected to deal only with the last form; thus we plot Re[I(f)] along the x axis, ±Im[I(f)] along the y axis, and $\log_{10}(f)$ along the z axis in a perspective representation of the resulting three-dimensional curve.[1] For some purposes, it may also be desirable to plot $\measuredangle I(f)$ in the x-y plane, $|I(f)|$ as a radius vector in the plane, and $\log_{10}(f)$ again along the z axis, but we have not investigated this possibility so far.

Figure 1 illustrates 3-D-perspective plotting of the impedance response of a simple electrical circuit. The heavy line is the 3-D curve and its three 2-D projections are shown in the appropriate planes. That appearing in the -Im(Z), Re(Z) plane is the conventional impedance-plane plot for the circuit. Clearly, the 3-D-perspective plot and its three projected curves show far more information than does any ordinary 2-D plot. We thus urge that 3-D-perspective plots become widely used.

Figures 2 and 3 show results for a slightly more complicated circuit. Unless otherwise indicated, the origin of the log(f) scale is taken at zero (f = 1 Hz), as are the origins of the other scales. Figure 3 is for a circuit where the two time constants are separated by a factor of 100 and the magnitudes of the two semicircular arcs by a factor of 10. Elsewhere[2] we have investigated the accurary and resolving power of nonlinear complex least squares fitting[3] in analyzing data containing random errors, especially for situations where time constants may not be well separated and/or where resistance ratios may be large. Excellent accuracy and resolution in estimating circuit parameters from somewhat inaccurate data is found for situations even more stringent than that of Fig. 3.

Figure 4 shows 3-D-perspective impedance and admittance plots for the ladder circuit at the top. The nominal values of its lumped-constant parameters were initially measured separately at a few frequencies (top values on circuit), then Y(f) was measured with a Solartron type 1172 response analyzer. The experimental arrangements have been described elsewhere.[4] Nonlinear complex least squares fitting[2] yielded a very good fit and the parameter estimates and their standard deviations shown in parentheses on the circuit. These values are probably more accurate than those obtained individually.

Finally, Fig. 5 shows results for a distributed β-PbF_2 sample at 474K. The circuit to which the admittance data were fitted by complex least squares is shown, along with the least squares parameter estimates.[2] Plotted on the 3-D-perspective impedance graph are both the original data points (solid circles) and points estimated from the fit (solid triangles, lying slightly above the data points at low frequencies). The fit was excellent and the parameters well defined by an objective procedure.

It should be emphasized that it will often be useful to show more than one 3-D-perspective

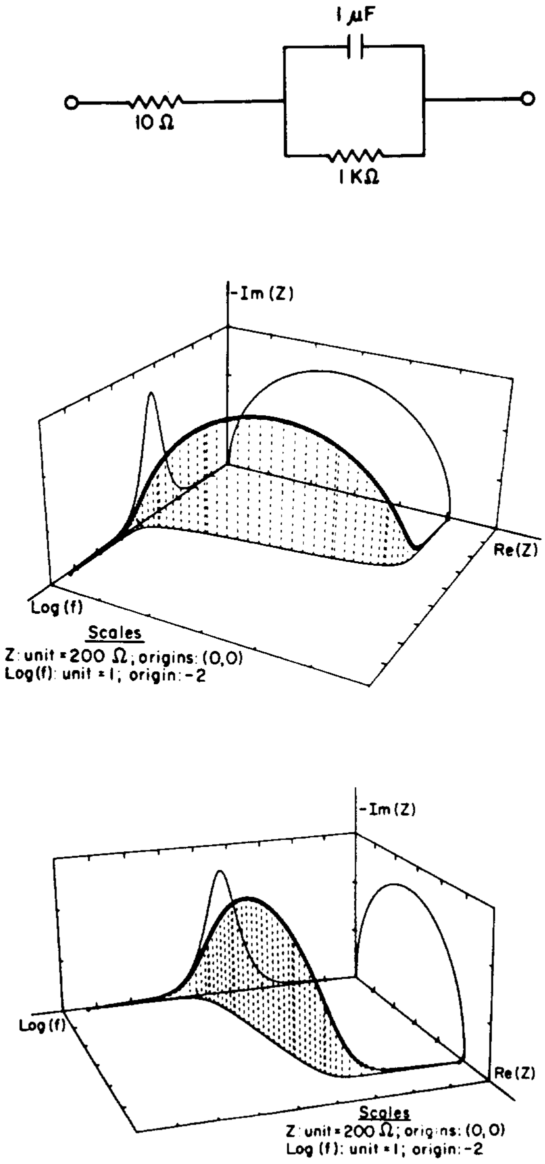

Figure 1: A simple equivalent circuit and two 3-D-perspective impedance plots viewed from different directions.

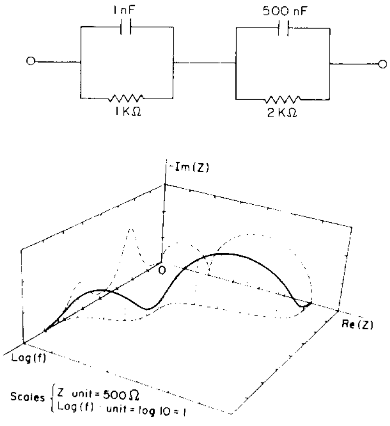

Figure 2: 3-D-perspective Z^* plot for a two-time-constant Voigt circuit.

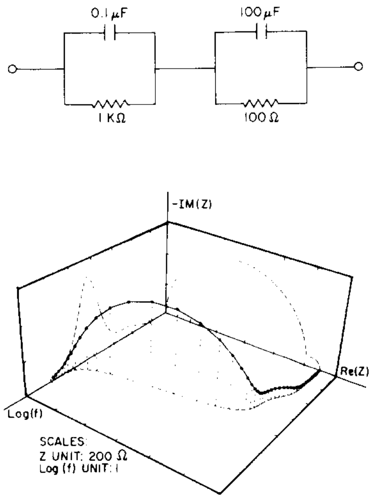

Figure 3: 3-D-perspective Z^* plot for a two-time-constant Voigt circuit.

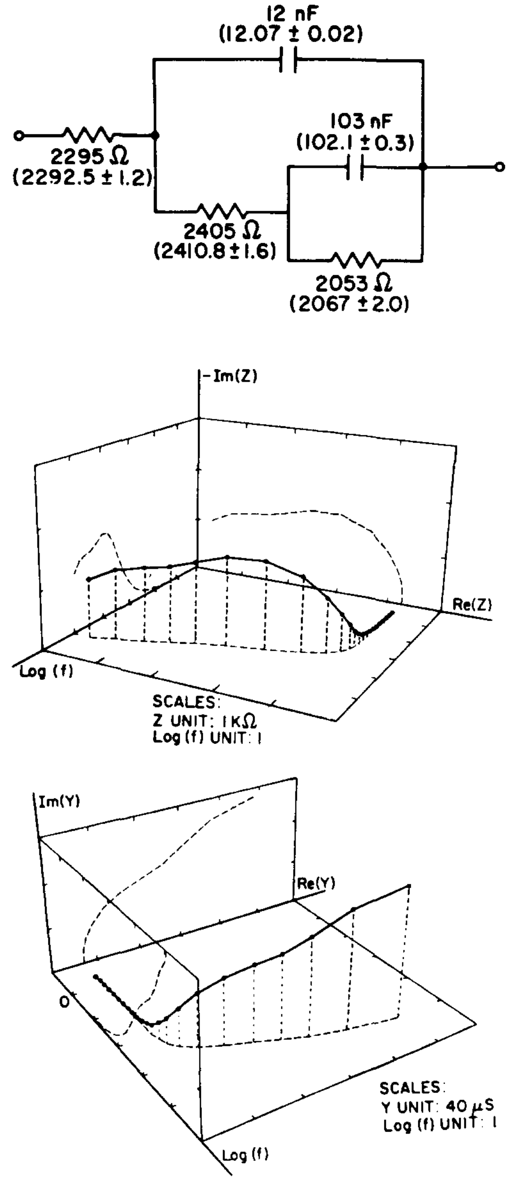

Figure 4: 3-D-perspective Z^* and Y plots for the lumped constant ladder circuit shown: nominal parameter values at top, fitted values in parentheses.

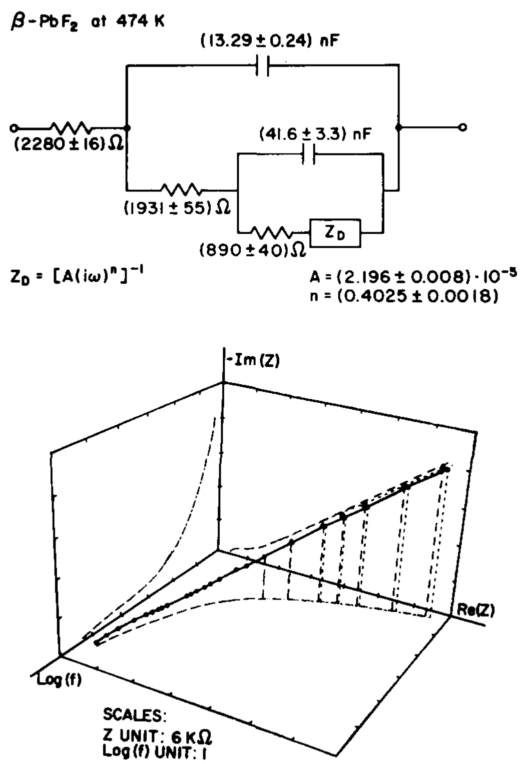

Figure 5: 3-D-perspective Z^* plot and fitted equivalent circuit for β-PbF_2 at 474K.

curve on the same graph. For example, if several sets of data are obtained for changes in a separate parameter (such as temperature, electrode separation, oxygen partial pressure, composition, or molarity), the unique surface formed by connecting the 3-D curves by isopleth lines will strikingly define the effects of the independent parameter variation. In some cases the independent parameter, such as temperature, might alternatively be used in place of frequency and plotted along the z axis, and frequency take its place in order to form a different surface.

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