

so much. It seems that this time we have to abandon the details of kinematics of densities resulting in the continuity equations, as it is obviously impossible to trace the currents inside the charged particles.

In spite of the lack of a continuity equation, the integral charge conservation law is guaranteed simply by the gauge invariance of the first kind. On the other hand, the gauge invariance of the second kind is by no means a necessary condition for the vanishing of the rest mass of the photon. The value zero for the photon rest mass may be secured by other devices, e.g. by a realistic compensation by means of charged bosons or by a formal mass renormalization which, in the frame of a convergent formalism, becomes a mathematically correct procedure. In this respect the new formalism does not constitute any drawback in comparison with the traditional quantum electrodynamics which was gauge invariant only formally, so that we were also obliged to renormalize the (infinite!) self mass of the photon.

ACKNOWLEDGMENT

The author is indebted to Professor M. Born for his kind interest in this work and for a stimulating discussion.

REFERENCES

- BLOCH, C., 1950, *K. Danske Vidensk. Selsk. Mat.-fys. Medd.*, **26**, 1
 BORN, M., and GREEN, H. S., 1949, *Proc. Roy. Soc. Edinb. A*, **92**, 470
 BORN, M., and PENG, H. W., 1944, *Proc. Roy. Soc. Edinb. A*, **62**, 40
 RAYSKI, J., 1950, *Acta Phys. Polon.*, **10**, 103; 1951, *Proc. Roy. Soc. A*, in the press.
 SCHWINGER, J., 1949, *Phys. Rev.*, **75**, 651.
 YANG, C. N., and FELDMAN, D., 1950, *Phys. Rev.*, **79**, 972 (see also KALLEN, G., 1950, *Ark. Fys.*, **2**, 371)
 YUKAWA, H., 1950, *Phys. Rev.*, **77**, 651.

Ferromagnetic Resonance and the Internal Field in Ferromagnetic Materials

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Communicated by J. H. E. Griffiths; MS. received 20th February 1951, and in amended form 22nd June 1951

ABSTRACT. A classical treatment of the domain energy terms of a homogeneous ferromagnetic solid leads to a formula for the internal field contributions from these terms. With this result, modifications in the resonance condition of ferromagnetic resonance arising from self energy, exchange energy, magnetocrystalline anisotropy, and applied or intrinsic stress are obtained and are applied to various crystalline anisotropy and stress conditions of interest in ferromagnetic resonance experiments. Finally, the bearing of the results on the anomalous g -values obtained in resonance experiments is considered.

§ 1. INTRODUCTION

THIS paper is chiefly concerned with the forces which influence the direction of the magnetization vector \mathbf{I} of a ferromagnetic substance in an external magnetic field and with the effects of these forces upon some of the phenomena of ferromagnetic resonance. A macroscopic viewpoint is adopted

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throughout, and all microscopic interactions are replaced by formulae for the magnetic potential energy density of the material which involve the macroscopic magnetization vector.

The usual case considered in calculating theoretical magnetization curves is that of an ellipsoidal ferromagnetic specimen in a static magnetic field (Stoner and Wohlfarth 1948); in this case the magnetization is uniform throughout the specimen and is time-independent. In ferromagnetic resonance experiments, however, a very small high-frequency oscillating magnetic field is applied to the ellipsoidal specimen, as well as a large static field oriented at right angles to the oscillating field. The magnetization of the body may then be neither space nor time independent, although, because of the relative magnitudes of the static and oscillating applied fields, variation of the magnetization vector with time and position will be small. One of the principal problems of ferromagnetic resonance, however, is the determination of this small space-and-time dependence of the direction and magnitude of the magnetization vector. In the purely static case the direction of the magnetization vector is determined from the general condition that the total magnetic potential energy of the body be a minimum. Because the magnetization is here uniform throughout the specimen, minimization of the expression for the magnetic energy density with respect to angle variables allows the direction of the magnetization vector to be uniquely specified.

In the present work the assumption will be made that the same general condition of minimum total magnetic potential energy can be applied to the oscillating field case when the magnetization of the body is homogeneous to yield the direction of the magnetization vector to a good order of approximation. When this minimization procedure is valid, the direction of the magnetization vector which is found is its instantaneous equilibrium direction \mathbf{I}_{eq} defined as the direction in which the magnetization vector would remain or reach quickly if the time variation of the oscillating field were suddenly halted at the instant under consideration (and if retarded fields were of negligible importance).

If the oscillating field is of very high frequency, the actual instantaneous direction of the magnetization vector \mathbf{I} may not coincide with the instantaneous equilibrium direction. The actual instantaneous direction and magnitude of the magnetization at a given point may be found, however, from the solution of the differential equation of motion satisfied by the magnetization vector; this equation involves the instantaneous internal, or local, field \mathbf{H} at the point in the body considered. The instantaneous equilibrium magnetization vector is, by definition, parallel to the instantaneous internal field, since minimum magnetic energy of the specimen results when the magnetization vector lies along the internal field at every point in the body.

In §2 the variational principle is applied to the total magnetic potential energy of a homogeneous non-conducting ellipsoidal ferromagnetic body to yield a general method for calculating the internal-field vector when an external magnetic field is applied, and it is shown how the instantaneous direction of the magnetization vector may be determined. The contributions to the internal magnetic field from the various magnetic potential energy terms present in a ferromagnetic specimen are evaluated in §3. In §§4-6 the influence on the ferromagnetic resonance condition of the various energy terms is discussed in detail for macroscopic single crystals and for polycrystalline aggregates. Finally, in §7 the bearing of the results of the paper upon the anomalous Landé g -factors found in resonance experiments is considered.

§ 2. CALCULATION OF THE INTERNAL FIELD

Before deriving a general expression for the internal field \mathbf{H}^i in terms of the magnetic potential energy density, it is desirable to define further quantities and to consider the dependence of these quantities upon space and time. Since the externally applied field \mathbf{H} is the sum of a static component \mathbf{H}_0 and a high-frequency oscillating component \mathbf{h} , the internal field \mathbf{H}^i will be a corresponding sum of \mathbf{H}_0^i and \mathbf{h}^i , where h/H_0 and h^i/H_0^i are very much smaller than unity. \mathbf{H}_0^i and \mathbf{h}^i will be independent of position within the body providing it is a non-conducting ellipsoid; if the ellipsoid is electrically conducting, h will decay with penetration depth because of the skin effect. Similarly, the magnetization vector \mathbf{I} will be a sum of a static component \mathbf{I}_0 and an oscillating part \mathbf{i} . Because of the smallness of the oscillating field \mathbf{h} compared with the static field, the oscillating component of the resulting magnetization will likewise be far smaller than the static component even at resonance, when the oscillating component is a maximum.

Finally, it is easy to show that the magnitude of the total magnetization vector is very nearly independent of time. The vector differential equation of motion of \mathbf{I} , at a given point in the body, is

$$\frac{1}{\gamma} \frac{\partial \mathbf{I}}{\partial t} = \mathbf{I} \times \mathbf{H}^i, \quad \dots (1)$$

where $\gamma = ge/2mc = g\mu_B/\hbar$ is the magnetomechanical ratio, g the Landé splitting factor, and μ_B the Bohr magneton. Note that the g -factor is assumed, subject to experimental verification, to be a constant independent of field orientation, body shape, etc. Equation (1) is just an expression for the rate of change of angular momentum per unit volume of the system. It shows that $\mathbf{I} \cdot \partial \mathbf{I} / \partial t = 0$ at all points in the magnetic body, and that therefore I is time-independent. However, a small additional term should be added to equation (1) in order to account for damping caused by interactions between spins in the magnetic material and between the spin system and the crystalline lattice (Kittel 1948, Bloembergen 1950). Such damping may destroy the time-independence of I , but since damping is usually found experimentally to be small, I still remains nearly time-independent; therefore the effect of damping will be neglected in the present treatment. It will now be shown how an expression for the internal field may be derived by extending a method used by Landau and Lifshitz (1935).

There are several contributions to the total magnetic potential energy density which may influence the direction of the magnetization vector in a ferromagnetic body. The following will be considered here: (a) the magnetic potential energy density of the body in the external magnetic field, (b) the potential energy density arising from the magnetic self-energy of the body associated with its shape, (c) the Heisenberg exchange, or Weiss, energy density, (d) the magneto-crystalline anisotropy energy density, which is thought mainly to arise from spin-orbit coupling between neighbouring atoms, and (e) the strain magnetic energy density connected with magnetostriction and dependent upon the state of strain of the body.

It should be noted that domain wall energy need not be explicitly considered in the present treatment because the static external magnetic field will always be taken sufficiently large for the specimen to be considered a single domain. The domain wall energies and thicknesses for zero applied fields have recently been considered by Lilley (1950) and by Kittel (1949b).

The magnetic potential energy density of a uniformly magnetized magnetic body (non-conducting ellipsoid) may usefully be considered as a function of the following variables*: $E = E(\mathbf{I}, X_i, \partial\mathbf{I}/\partial X_i)$, where X_i represents the three rectangular coordinates. Hence the total potential energy of the body will be

$$\int_V E\left(\mathbf{I}, X_i, \frac{\partial\mathbf{I}}{\partial X_i}\right) dV.$$

Applying the condition that the total magnetic potential energy of the body must be stationary with respect to small variations of the magnetization vector \mathbf{I} , one obtains

$$\int_V \delta E\left(\mathbf{I}, X_i, \frac{\partial\mathbf{I}}{\partial X_i}\right) dV = \int_V \left\{ \frac{\partial E}{\partial \mathbf{I}} - \sum_i \frac{\partial}{\partial X_i} \left[\frac{\partial E}{\partial (\partial \mathbf{I} / \partial X_i)} \right] \right\} \cdot \mathbf{I} dV = 0.$$

Now the magnitude of \mathbf{I} has been assumed to be a constant independent of time. Therefore the variation $\delta\mathbf{I}$ is not completely arbitrary but must lie in the plane perpendicular to \mathbf{I}_{eq} . This is because the variation consists of small changes in the direction of \mathbf{I} from the instantaneous equilibrium direction \mathbf{I}_{eq} . Since the variation is otherwise arbitrary in magnitude and direction, however, the integrand of the above integral must itself be zero. One thus obtains

$$(\mathfrak{H}E) \cdot (\delta\mathbf{I}_{eq}) = 0, \tag{2}$$

where \mathfrak{H} is the differential vector operator

$$\left\{ -\frac{\partial}{\partial \mathbf{I}} + \sum_i \frac{\partial}{\partial X_i} \left[\frac{\partial}{\partial (\partial \mathbf{I} / \partial X_i)} \right] \right\}.$$

Equation (2) can only be satisfied if the vector $\mathfrak{H}E$ is parallel or antiparallel to \mathbf{I}_{eq} or is zero, since $\delta\mathbf{I}_{eq}$ is perpendicular to \mathbf{I}_{eq} . However, $\mathfrak{H}E$ cannot, in general, be zero as long as an external field is applied. Further, by definition, \mathbf{I}_{eq} is always parallel to the internal field \mathbf{H}^i . Therefore $(\mathfrak{H}E)$ must be equal to \mathbf{H}^i apart from a proportionality factor. This factor can be shown to be unity by applying the operator \mathfrak{H} to an energy density term for which the corresponding contribution to the internal field is already known. Hence

$$\mathbf{H}^i = (\mathfrak{H}E) \tag{3}$$

and the instantaneous equilibrium direction of the magnetization vector at any point in the body may be determined from the condition that

$$\mathbf{I}_{eq} \times \mathbf{H}^i = 0. \tag{4}$$

Since \mathbf{H}^i itself will usually be a function of \mathbf{I} and its individual components, equation (4) must be used to determine the actual equilibrium direction of \mathbf{I} . A help in solving this vector equation is the auxiliary equation $\mathbf{I}_0 \times \mathbf{H}_0^i = 0$, which expresses the fact that the static component of the magnetization lies in the direction of the static component of the internal field. Equation (3) will be used later to compute some of the contributions of the different energy densities to the internal field.

When the applied frequency is very high, the instantaneous direction of the magnetization vector will not coincide with its instantaneous equilibrium direction. The actual instantaneous direction may be found using the equation of motion (equation (1)), which may be rewritten as

$$\frac{1}{\gamma} \frac{\partial \mathbf{I}}{\partial t} = \frac{1}{\gamma} \frac{\partial \mathbf{i}}{\partial t} = \mathbf{I} \times \mathbf{H}^i = (\mathbf{I}_0 + \mathbf{i}) \times (\mathbf{H}_0^i + \mathbf{h}^i) \simeq \mathbf{I}_0 \times \mathbf{h}^i + \mathbf{i} \times \mathbf{H}_0^i \tag{5}$$

* The index here and in all further equations takes on values from 1 to 3.

since $\mathbf{i} \times \mathbf{h}'$ is a very small second-order term and may be neglected. Because $\gamma = 1.7 \times 10^7$ cycles/sec/oersted, the left side of equation (5) is very small unless the applied frequency is high. When the frequency is low $\mathbf{I} \times \mathbf{H}' \approx 0$, and the instantaneous equilibrium and actual directions of the magnetization vector are always effectively the same. But when the frequency is high this is no longer the case, and the direction of \mathbf{i} must be found by solving equation (5). Since \mathbf{I}_0 is effectively the saturation value of the magnetization and its direction is fixed by the relation $\mathbf{I}_0 \times \mathbf{H}_0' = 0$, the instantaneous direction of the total magnetization vector $\mathbf{I} = \mathbf{I}_0 + \mathbf{i}$ may then be found.

§ 3. INTERNAL FIELD CONTRIBUTIONS

In this section the individual contributions to the total internal field from the five potential energy densities mentioned in § 2 will be calculated using equation (3). The five energy density terms may be written as functions of the magnetization vector as follows:

(a) External-field energy density: $E_{\text{ext}} = -\mathbf{H} \cdot \mathbf{I}$.

(b) Self-energy density (for a non-conducting ellipsoid only): $E_s = \frac{1}{2} \sum N_j^s I_j^2$.

(c) Exchange energy density (Stoner and Wohlfarth 1948)*:

$$\begin{aligned} E_{\text{exc}} &= -\frac{1}{2} N_w \mathbf{I} \cdot \mathbf{I} + \frac{1}{2} C \sum [(\nabla I_i) \cdot (\nabla I_i)] \\ &= -\frac{1}{2} N_w \mathbf{I} \cdot \mathbf{I} + \frac{1}{2} C \sum \left[\frac{\partial \mathbf{I}}{\partial X_i} \cdot \frac{\partial \mathbf{I}}{\partial X_i} \right]. \end{aligned}$$

(d) Crystalline anisotropy energy for single crystals (Becker and Doring 1939):

$$\text{Hexagonal crystals: } E_c = K_1'(1 - \alpha_1^2) + K_2'(1 - \alpha_2^2).$$

$$\begin{aligned} \text{Cubic crystals: } E_c &= K_1[\alpha_1^2 \alpha_2^2 + \alpha_2^2 \alpha_3^2 + \alpha_3^2 \alpha_1^2] + K_2[\alpha_1^2 \alpha_2^2 \alpha_3^2] \\ &= \frac{1}{2} K_1[1 - (\alpha_1^4 + \alpha_2^4 + \alpha_3^4)] + K_2[\alpha_1^2 \alpha_2^2 \alpha_3^2]. \end{aligned}$$

(e) Strain energy density for cubic single crystals (Becker and Doring 1939)†:

$$E_{\text{st}} = -\frac{3}{2} \lambda_{100} \sum \alpha_i^2 P_{ii} - 3 \lambda_{111} \{ \alpha_1 \alpha_2 P_{12} + \alpha_2 \alpha_3 P_{23} + \alpha_3 \alpha_1 P_{31} \}.$$

In the above, α_1 , α_2 and α_3 are the direction cosines of the magnetization vector with respect to the X_1 , X_2 , X_3 axes, taken along the cubic axes of the cubic crystal and taken with the X_1 axis in the direction of the principal crystalline axis of the hexagonal crystal, so that $\alpha_1 = I_1/I = (\mathbf{I} \cdot \mathbf{i}_1)/I$, where \mathbf{i}_1 is a unit vector along the X_1 axis, etc. The P_{ij} 's are the components of the stress tensor referred to the same coordinate system. K_1' , K_2' and K_1 , K_2 are crystalline anisotropy constants which may be determined experimentally. Recent values of these constants are quoted by Kittel (1949 b). The N_j^s 's are the shape demagnetization constants applying when the coordinate system is taken along principal axes of the ellipsoidal specimen. N_w is the Weiss molecular field constant, and C is a constant approximately equal to $\frac{1}{3} a^2 N_w$, where a is the grating spacing of the material. The results of more accurate calculations of the quantity C are given by Lifshitz (1944) and by Stoner and Wohlfarth (1948). Finally, λ_{100} and λ_{111} are the saturation values of the magnetostriction constants of the ferromagnetic crystal in the [100] and [111] directions respectively. Numerical values of these constants for iron and nickel are given by Becker and Doring (1939, pp. 277-280).

* This formula has been derived only for substances having cubic structure.

† This expression is valid only for the region of elastic strain and for stress tensor elements independent of position (homogeneous stress).

Since the total energy density is to a very good approximation the sum of the individual energy density terms, the contribution to the internal field of each energy density term may be obtained by applying the operator \mathfrak{H} to each term in turn. One then finds the following results:

$$\mathbf{H}_{\text{ext}}^i = \mathbf{H} \equiv \mathbf{H}_0 + \mathbf{h} \quad \dots\dots(6)$$

$$(H_{\text{st}}^i)_j = - \sum N_j^s I_j \quad \dots\dots(7)$$

$$\mathbf{H}_{\text{vib}}^i = N_w \mathbf{I} + C \nabla^2 \mathbf{I} \quad \dots\dots(8)$$

$$\mathbf{H}_c^i = \left[\frac{2K_1'}{I} + \frac{4K_2'}{I} \left\{ 1 - \left(\frac{I_1}{I} \right)^2 \right\} \right] \left(\frac{I_1}{I} \right) \mathbf{i}_1 \quad \text{Hexagonal} \quad \dots\dots(9)$$

$$(H_c^i)_j = \frac{2K_1}{I^4} I_j^3 - \frac{2K_2}{I^6} I_j I_k^2 I_l^2 \quad (j, k, l = 1, 2, 3 \text{ and permutations})$$

Cubic \dots\dots(10)

$$(H_{\text{st}}^i)_j = \frac{3}{I^2} \sum_k [(\lambda_{100} - \lambda_{111}) \delta_{jk} + \lambda_{111}] P_{jk} I_k \quad \dots\dots(11)$$

Some of these equations are written directly in terms of vectors, others in terms of the *j*th component of the corresponding vector, in the interest of simplicity.

From the above expressions the direction and magnitude of each contribution to the total internal field may be obtained. Previously, Bozorth (1949) obtained the approximate maximum magnitude of the internal field contributions arising from several of the energy density terms considered here by equating the terms separately to $\mathbf{H}^i \cdot \mathbf{I}$. Further, Kittel (1948) computed the quantity $\mathbf{I} \times \mathbf{H}_c^i$ for the case of crystalline anisotropy for several special directions and planes in single crystals. Finally, Brown (1940) has used a different variational approach to obtain general equations equivalent to $(\mathbf{I}_0 \times \mathbf{H}_0^i)_j = 0$, expressing the balance of torques on the magnetization vector when no oscillating field is applied, and including the static effect of all the potential energy terms considered here but with these terms expressed somewhat differently. The present results, however, give both the magnitude and the general directional dependence of the various static and oscillating internal field contributions in convenient form. It should be noted, however, that the internal field contributions due to crystalline anisotropy and strain in cubic crystals are not unique, because the relation $\alpha_1^2 + \alpha_2^2 + \alpha_3^2 = 1$ may be used to transform the expressions for the energy densities to different forms from those used here. Nevertheless, such indeterminacy does not affect the term $\mathbf{I} \times \mathbf{H}^i$ which determines the direction of \mathbf{I} ; therefore both the direction of \mathbf{I} and the effect of the above energy terms upon ferromagnetic resonance phenomena remain unique.

§ 4. FERROMAGNETIC RESONANCE EXPERIMENTS

The foregoing internal field contributions may be used to compute the dependence of the magnetization curves of ferromagnetic single crystals along different crystal directions upon ellipsoid shape, crystalline anisotropy and applied stress. Part of such a programme has been carried out by Stoner and Wohlfarth (1948), who, however, used the method of direct minimization of the total magnetic energy density to obtain the dependence of the magnetization components upon an applied static magnetic field. In solving such a problem, the actual internal field contributions derived here are unnecessary, although they shed additional light on the physical situation considered. In considering, however, the influence

of ellipsoid shape, exchange anisotropy, crystalline anisotropy and strain upon ferromagnetic resonance phenomena, explicit expressions for the internal field contributions are useful.

In a ferromagnetic resonance experiment two external magnetic fields are applied to an ellipsoidal ferromagnetic body. The specimen may be a single crystal or polycrystalline, non-conducting or conducting. The fields are usually taken along two mutually perpendicular principal axes of the ellipsoid. One of the fields is large and static but can be varied in magnitude over a wide range; the other is produced by incident microwave electromagnetic radiation and is very small in magnitude. In making ferromagnetic resonance experiments, the frequency of the incident radiation is commonly held constant and the magnitude of the static field varied. It is found that a maximum of the absorption of microwave power in the specimen occurs for a unique static field strength. This resonant field strength, H_0^r , is that field strength which makes the effective Larmor frequency of the spins of the ferromagnetic material, $(\gamma/2\pi)H^{\text{eff}}$, equal to the frequency of the applied radiation ν . Since the effective field H^{eff} involves all the internal field contributions, theoretical determination of the resonant field strength requires information concerning such contributions.

It is usually assumed in ferromagnetic resonance calculations that the externally applied static field is always of sufficient strength to cause the resulting static component of the total magnetization vector to be substantially equal to the saturation magnetization of the material and to lie in the direction of the static field. Crystalline anisotropy and applied stress may tend to pull the static magnetization vector away from the field direction, but if the field is sufficiently strong (as is usually the case experimentally) such effects may be neglected. Then any influence of shape, crystalline anisotropy, or strain on the resonance phenomena arises from the interaction between their internal field contributions and the very weak oscillating component of the magnetization vector produced by the oscillating magnetic field of the incident radiation.

There are two particularly important quantities which may be obtained from the results of a ferromagnetic resonance experiment. These are the Landé g -factor determined from the resonant magnetic field strength, and the damping factor which may be calculated from the shape of the microwave absorption resonance curve. The importance of these two factors in giving insight into the coupling between ferromagnetic electrons and between these electrons and the crystal field has been recently discussed by Kittel (1949 a) and Van Vleck (1950). In the present treatment we are especially concerned with the calculation of the Landé g -factor for different ferromagnetic specimen shapes, orientation of applied fields, crystalline nature and applied stress, although other quantities, such as crystalline-anisotropy and magnetostriction constants, may also be obtained from the analysis of resonance experiments. It is not expected that the g -factor of a given ferromagnetic material should depend appreciably upon any of the above conditions; however, its calculation requires knowledge of the effects of these factors upon the resonant magnetic field. Such knowledge may be embodied in the explicit form of H^{eff} , and, given an experimental H_0^r for a specific microwave frequency, the corresponding g -value may be computed from the resonance condition $\nu = (ge/4\pi mc)H^{\text{eff}}$. The presence in an experimental material of any anisotropy which is not recognized by an appropriate modification of H^{eff} can obviously cause an incorrect value of g to be obtained from the experiment.

§ 5. THE GENERAL RESONANCE CONDITION

The ferromagnetic resonance condition may be obtained from the solution of the equation of motion (equation (5)) when the explicit form of the internal field components is known. The equation of motion may be solved in any convenient coordinate system; the most convenient is often the rectangular system (X_1, X_2, X_3) , defined by coincident ellipsoid and single-crystal principal axes. For the present work it is useful to introduce another rectangular coordinate system (X_1', X_2', X_3') rotated with respect to the (X_1, X_2, X_3) axis with the static applied field H_0 always taken along the X_3' axis and the oscillating field h along the X_1' axis. By solving the equation of motion in this primed system, general resonance conditions applicable to arbitrary orientations of the applied fields with respect to ellipsoid principal axes, single crystal axes, or an applied or intrinsic stress system may be obtained immediately.

In order to solve the equation of motion in the primed coordinate system, the internal field vector must be resolved in the primed system. The internal field contributions given in equations (7), (9), (10), and (11) involve magnetization vector components referred to the unprimed system; the transformation of the internal field expressions and the magnetization vector components to the primed system is straightforward and is accomplished by introducing the nine direction cosines γ_{ij} connecting an X_i' axis to an X_j axis. One finds that the internal field components in the primed system due to shape effects, crystalline anisotropy, or strain may each be written in the general form

$$(H_a)_j' = \sum_k N_{jk}^a I_k', \quad \dots\dots(12)$$

where the index 'a' denotes anisotropy due to shape, single-crystal structure, or strain. The N_{jk}^a matrix elements are effective demagnetization constants due to anisotropy referred to the primed system; they will, in general, involve the γ_{ij} 's.

The equation of motion for arbitrary N_{jk}^a values may now be solved using equation (12). The components of the over-all internal field, including applied fields, are of the form

$$(H^i)_j' = H_j' - \sum_k N_{jk}^a I_k', \quad \dots\dots(13)$$

where $H_1' = h, H_2' = 0, H_3' = H_0$. Since the static field H_0 is taken as far greater than any static anisotropy field, the static magnetization vector will lie very nearly along the direction of the static applied field. In this case, the components of the magnetization vector are: $I_1' = i_1', I_2' = i_2', I_3' = I_0$. The solution of the equation of motion with the above form of H^i' was first carried out by Kittel (1947, 1948). In Kittel's analysis the $[N_{jk}^a]$ matrix was diagonal and the following expression for H^{eff} was obtained:

$$H^{\text{eff}} = \{[H_0^i + (N_{22}^a - N_{33}^a)I_0][H_0^r + (N_{11}^a - N_{33}^a)I_0]\}^{1/2}, \quad \dots\dots(14)$$

where H_0^i is the resonant value of the static field.

It turns out that the same result is an excellent approximation when the $[N_{jk}^a]$ matrix is not diagonal, provided that H_0 is large enough to make I_0 lie along its direction. In the general case of arbitrary orientation between applied fields and crystal or ellipsoid axes, the off-diagonal elements of $[N_{jk}^a]$ are not, in fact, zero. Nevertheless, the same resonance equation applies very closely. In order to compute the resonance shifts due to the various different sources of anisotropy, it is thus only necessary to calculate the diagonal elements of the corresponding $[N_{jk}^a]$ matrices. When two or more sources of anisotropy are

present in a specimen to be used in a resonance experiment, the matrix elements occurring in the resonance condition are the sums of the matrix elements arising from the individual anisotropies, as can readily be seen from the linearity of equation (5) in \mathbf{H} and of equation (12) in N_{ij}^a .

In the Appendix general expressions for the $(N_{11}^a - N_{33}^a)$ and $(N_{22}^a - N_{33}^a)$ factors occurring in the resonance condition are given for shape, crystalline anisotropy and strain anisotropy contributions. Special cases of these expressions for situations which have arisen or might be important in resonance experiments are considered in the following section.

§6. ANISOTROPY CONTRIBUTIONS TO THE RESONANCE CONDITION

(i) Shape

Ferromagnetic resonance experiments have thus far been carried out with spherical magnetic specimens of high resistivity (Hewitt 1948, Bickford 1950, Yager *et al.* 1950) and with either high resistivity ferritic or conducting metallic disc-shaped specimens (Griffiths 1951). In these cases the applied magnetic fields are always taken along principal ellipsoid axes (considering a thin disc as an oblate spheroid) and the general treatment in the Appendix is unnecessary; the results given in equations (23) are only required when field directions and principal axes do not coincide. The shape effects derived below apply both to polycrystalline and to single crystal materials.

The effective fields for the above specimen shapes are easily obtained by substituting the shape demagnetization constants N_j^s directly for the N_{jj}^a elements occurring in equation (14). For a non-conducting isotropic sphere $N_1^s = N_2^s = N_3^s = 4\pi/3$ and H_{eff} reduces to H_0^1 . There is no shape anisotropy here and hence no resonance shift. For a non-conducting thin disc the demagnetization constants for fields lying in the plane of the disc are approximately π^2/m , while that for a field normal to the disc is $(4\pi - 2\pi^2/m)$, where m is the ratio of disc diameter to thickness (Osborn 1945, Stoner 1945). There are two field orientations of experimental interest, both applied fields lying in the plane of the disc, and static field perpendicular to the disc. The demagnetization constants in these two cases are, respectively, $N_1 = N_3 = \pi^2/m$, $N_2 = 4\pi - 2\pi^2/m$, and $N_1 = N_2 = \pi^2/m$, $N_3 = 4\pi - 2\pi^2/m$. The corresponding effective fields are

$$H_{\parallel}^{\text{eff}} = \left[\left(B_0^r - \frac{3\pi^2}{m} I_0 \right) H_0^r \right]^{1/2}; \quad H_{\perp}^{\text{eff}} = \left[H_0^1 - \left(4\pi - \frac{3\pi^2}{m} \right) I_0 \right]$$

(non-conducting discs). (15)

When the resistivity of a specimen is not so high that it may be considered as virtually non-conducting, the skin effects begin to become important. Because of this effect, the magnetic fields in the specimen are inhomogeneous since there is a decay of the amplitudes of the oscillating components of the field as they penetrate into the disc. In this case the expression for the magnetic potential energy should contain terms arising from and depending upon the distribution of $\nabla \cdot \mathbf{I}$, which would cause the simple expression for E_s given in §2 to be incorrect. A good approximation to the correct value of the effective field in this case may be obtained, however, by remembering that at the high frequencies employed the skin depth is always considerably less than the diameter or thickness of the disc. Then the demagnetization constants in the direction of oscillating field

components have approximately the values they would have if the disc were of infinite diameter. Therefore, for parallel fields, $N_1=0$, $N_2=4\pi$, $N_3=\pi^2/m$, while for perpendicular static fields, $N_1=0$, $N_2=0$, $N_3=4\pi-2\pi^2/m$. The effective field expressions are

$$H_{\parallel}^{\text{eff}} = \left[\left(B_0^r - \frac{\pi^2}{m} I_0 \right) \left(H_0^r - \frac{\pi^2}{m} I_0 \right) \right]^{1/2}; \quad H_{\perp}^{\text{eff}} = \left[H_0^r - \left(4\pi - \frac{2\pi^2}{m} \right) I_0 \right]$$

(conducting discs). (16)

Experiments designed to compare g_{sphere} with g_{disc} (parallel field orientation) for a polycrystalline ferrite of high resistivity were performed by Hewitt (1948), who obtained reasonably good agreement between the g -values, indicating that the resonance formulae for these cases are at least good approximations. Further, an experiment comparing g_{\parallel} and g_{\perp} for a supermalloy disc specimen was carried out by Kittel, Yager and Merritt (1949) who, found a difference between g_{\parallel} and g_{\perp} of 2.5%. Using the above expression for H_{\perp}^{eff} , which is slightly more accurate than that used by Kittel *et al.*, one obtains a difference of approximately 1.1%; this can probably be ascribed to inexact knowledge of I_0 .

(ii) Exchange

In order to evaluate the effect of exchange anisotropy on the resonance condition it is necessary to add the exchange internal field contribution given in equation (8) to the equation of motion and solve it with this added term. If the magnetic material is non-conducting, however, the contribution will be zero since $\nabla^2 \mathbf{I}$ will then be zero and $\mathbf{I} \times N_{\text{ex}} \mathbf{I}$ is identically zero. Because of the skin effect present when the material is conducting, $\nabla^2 \mathbf{I}$ is not zero and becomes $\nabla^2 \mathbf{i}$. Exact solution of the equation of motion with such a term taken in conjunction with Maxwell's equations is a lengthy complicated process since the material becomes triply refracting to microwaves. The results of such a solution by the author (Macdonald 1950) concur with those of a perturbation treatment by Kittel and Herring (1950) in indicating no appreciable resonance shift from this cause for metals at room temperature.

(ii) Magnetocrystalline Anisotropy

Thus far no resonance experiments have been reported on hexagonal-close-packed single crystals. Experiments to determine the anisotropy constants K_1' and K_2' might be carried out with the static magnetic field always in a plane perpendicular to the principal crystal axis or, alternatively, with the static field lying in a plane containing the crystal axis. If θ is the angle between the static field and the crystal axis, the direction cosines in these two cases are, respectively, $\gamma_{21}=1$, $\gamma_{11}=\gamma_{31}=0$ and $\gamma_{11}=\sin \theta$, $\gamma_{31}=\cos \theta$, $\gamma_{21}=0$. The corresponding contributions from equations (24) (see Appendix) to the effective field are

$$(N_{11}^c - N_{33}^c)I_0 = 0; \quad (N_{22}^c - N_{33}^c)I_0 = -\frac{2}{I_0} (K_1' + 2K_2') \quad \dots \dots (17)$$

and $(N_{11}^c - N_{33}^c)I_0 = \frac{2K_1'}{I_0} \cos 2\theta + \frac{4K_2'}{I_0} \sin^2 \theta (1 + 2 \cos 2\theta);$

$$(N_{22}^c - N_{33}^c)I_0 = \frac{2K_1'}{I_0} \cos^2 \theta + \frac{K_2'}{I_0} \sin^2 2\theta. \quad \dots \dots (18)$$

When $\theta=0$ the last two equations reduce to a result first obtained by Kittel (1948), who considered terms in K_1' only. For cobalt the factors $2K_1'/I_0$ and K_2'/I_0 are approximately 6,000 and 700 gauss respectively, showing that crystalline anisotropy can affect the resonance condition greatly for this material.

Two interesting experiments have been carried out on cubic single crystals. Kip and Arnold (1949) investigated the dependence of the resonant field for a silicon-iron single crystal disc upon the angle θ between the static field and the [001] crystal direction when the static field was constrained to lie in the (010) plane. In this case the non-zero direction cosines are $\gamma_{11}=\gamma_{33}=\cos\theta$, $\gamma_{13}=-\gamma_{31}=\sin\theta$ and $\gamma_{22}=1$. The contributions to the effective field from equations (25) are

$$(N_{11}^c - N_{33}^c)I_0 = \frac{2K_1}{I_0} \cos 4\theta; \quad (N_{22}^c - N_{33}^c)I_0 = \frac{K_1}{2I_0} (3 + \cos 4\theta) + \frac{K_2}{2I_0} \sin^2 2\theta. \quad \dots (19)$$

These results were first obtained by Kittel (1948, 1949 a). Kip and Arnold found the expected dependence of H_0^1 upon θ at constant applied frequency and obtained a good value of K_1 for silicon-iron as well as a g -value independent of θ .

Recently Yager, Galt, Merritt and Wood (1950) investigated the effect of rotating the static field applied to a single-crystal ferrite sphere in a (110) crystal plane. If θ is the angle between the applied field and the [001] direction, the non-zero direction cosines applying to this case are

$$\gamma_{11}=\gamma_{12}=\sqrt{(1/2)}\gamma_{33}=\sqrt{(1/2)}\cos\theta, \quad \gamma_{31}=\gamma_{32}=-\sqrt{(1/2)}\gamma_{13}=\sqrt{(1/2)}\sin\theta.$$

The general equation (25) then reduces to

$$\left. \begin{aligned} (N_{11}^c - N_{33}^c)I_0 &= \frac{K_1}{I_0} (2 - \sin^2\theta - 3\sin^2 2\theta) + \frac{K_2}{16I_0} \sin^2\theta (1 - 60\cos^2\theta + 65\cos^4\theta); \\ (N_{22}^c - N_{33}^c)I_0 &= \frac{2K_1}{I_0} \left(1 - 2\sin^2\theta - \frac{3}{8}\sin^2 2\theta \right) - \frac{K_2}{16I_0} \sin^2 2\theta (7 - 5\cos^2\theta). \end{aligned} \right\} \quad \dots (20)$$

The first-order terms in these equations have been reported earlier by Bickford (1950). Yager *et al.* found the θ -dependence indicated by the above equations and an excellent value of K_1 for the ferrite investigated: the K_2 terms were apparently negligible for this ferrite.

(iv) Strain

From the general equations (26) given in the Appendix it is relatively easy to compute the strain contribution to the resonance condition when the applied fields lie in a (010) or (110) crystal plane and none of the stress tensor elements is zero. Here, however, we shall consider some even simpler situations of more experimental interest.

In the case of uniform pressure P applied to a ferromagnetic ellipsoid, the stress tensor elements are given by $P_{ij} = -P\delta_{ij}$, and there is no contribution from the general equations (26) because there is no strain anisotropy. This is not the case, however, for plane stress even in the simplest and experimentally most interesting case of a purely radial stress T_0 (T_0 is positive for tension, negative for compression) applied at the circumference of a circular disc. Here the only

non-zero stress tensor elements are (for a disc with its normal in the [010] direction) $P_{11} = P_{33} = T_0$. On substituting these results into the relations (26), and simplifying, one obtains

$$(N_{11}^{st} - N_{33}^{st})I_0 = \frac{3\lambda_{100}T_0}{I_0}(\gamma_{12}^2 - \gamma_{32}^2); \quad (N_{22}^{st} - N_{33}^{st})I_0 = \frac{3\lambda_{100}T_0}{I_0}(\gamma_{22}^2 - \gamma_{32}^2). \dots\dots(21)$$

In an experiment to measure the effect of applied stress of this character, produced, for example, by means of a draw-strap around the disc, the applied magnetic fields would be taken either in the (010) crystal plane (the plane of the disc) or with the static field perpendicular to the disc. For these two conditions the above equations reduce to

$$\left. \begin{aligned} \text{Parallel fields: } (N_{11}^{st} - N_{33}^{st})I_0 &= 0; \quad (N_{22}^{st} - N_{33}^{st})I_0 = 3\lambda_{100}T_0/I_0, \\ \text{Perpendicular static field: } (N_{11}^{st} - N_{33}^{st})I_0 &= (N_{22}^{st} - N_{33}^{st})I_0 = \frac{-3\lambda_{100}T_0}{I_0}. \end{aligned} \right\} \dots\dots(22)$$

In neither case is the contribution to the effective field zero, although rotation of the fields around the normal to the disc has no effect (crystalline anisotropy neglected) since there is no strain anisotropy in the plane of the disc. These non-zero results are somewhat unexpected and are discussed later in connection with actual experiments.

Another simple stress system of experimental interest consists of a unidirectional stress T applied in the $[\beta_1, \beta_2, \beta_3]$ crystallographic direction. Here the stress elements are $P_{ij} = \beta_i\beta_jT$. In the case where the applied fields lie in the plane of a single-crystal disc cut so that this plane coincides with the (010) crystallographic plane, and where the unidirectional stress is also applied in this plane, both the magnetostriction constants λ_{100} and λ_{111} may be obtained by measuring the change of H_0^2 as the fields are rotated with respect to the crystal axes, keeping the stress direction fixed with respect to these axes or, alternatively, keeping the stress direction fixed and shifting the stress direction with respect to the crystal axes. No experiments of this type have been reported as yet.

The above strain contributions have all applied to cubic ferromagnetic single crystals. Ferromagnetic resonance experiments are often made, however, on polycrystalline materials in which the individual crystallites have approximately random orientation. The extension of the strain formulae to isotropic polycrystalline material may be made in two different ways. In the first method the complete resonance condition, including self-energy, crystalline anisotropy, and strain contributions, must be averaged over all possible directions of the crystallite axes, preferably taking into account absorption line broadening due to relaxation processes, etc. Since this would be extremely complicated to carry out, even if crystalline shape and size distributions were known, it is simpler to make the approximation of isotropic magnetostriction usual in dealing with polycrystalline aggregates of ferromagnetic crystallites. This approximation consists of taking $\lambda_{100} = \lambda_{111} = \lambda$. The effective strain demagnetization constants in equation (26) then simplify to give

$$(N_{11}^{st} - N_{33}^{st})I_0 = \frac{3\lambda}{I_0}(P_{33}' - P_{11}'); \quad (N_{22}^{st} - N_{33}^{st})I_0 = \frac{3\lambda}{I_0}(P_{33}' - P_{22}'),$$

where the primed stress elements are referred now to the primed (field) coordinate system.

When isotropic stress T_0 is applied in the plane of a polycrystalline disc, the effective field contributions are given by equations (22) with λ_{100} replaced by λ . The sign of these factors depends both upon the sign of λ and upon whether the applied stress is tensile or compressive. The change in the resonance field caused by these factors may be quite appreciable; for example, Kittel (1949 b) gives for $3\lambda T_0/I_0$ the values 600, 600, 4,000 gauss for iron, cobalt and nickel, respectively, when T_0 is taken as the breaking stress of the material.

No resonance experiments deliberately undertaken to observe stress effects on single crystals or polycrystalline materials have been published as yet. However, the resonant field dependence on applied plane stress has been verified by Macdonald (1950, 1951 a, b)*, who investigated resonance shifts for thin nickel films evaporated on mica. Here, applied isotropic plane stress (independently determined from magnetometer measurements) was found to be produced by differential contraction between mica and nickel on cooling after evaporation. Such stress was dependent on film thickness and produced an apparent dependence of the g -factor on film thickness. No dependence of the g -factor on applied stress was observed, however, when the stress-corrected resonance condition was used. In the course of making resonance measurements from room temperature to the Curie point on a nickel disc silver-soldered to a copper block, Standley (unpublished) observed a decrease in the apparent g -value with increasing temperature. Calculations by the author (Macdonald 1950) have shown that such decrease was only apparent, being caused by the temperature-dependent plane tension produced in the nickel disc by the greater expansion on heating of copper as compared with nickel. It may be postulated that a similar small decrease in g with increasing temperature for nickel found by Bloembergen (1950) and reported to be within experimental error was caused by the same phenomenon. Further, the 4% decrease in g with decreasing temperature over the range from room temperature to -153°C . found by Bickford (1950) for a Fe_3O_4 single-crystal disc may be susceptible to the same general explanation, although the magnetostriction constant for magnetite may depend significantly upon temperature in this region. The dependence of the resonant field upon unidirectional stress applied to a polycrystalline specimen has not been verified in detail experimentally so far, although Macdonald (1950) observed some directional dependence of H_0^{r} upon the angle between the applied static field in the plane of an evaporated nickel disc and the coplanar direction of a magnetic field applied during annealing of the specimen.

§ 7. CONCLUSION

In all the resonance experiments discussed in § 6 the g -values obtained were substantially greater than 2.00, the value to be expected for free electron spins. In fact the great majority of all g -values measured so far have significantly exceeded the free electron value, usually lying in the range between 2.1 and 2.3. One of the objects of the present work was to investigate fully the modifications in the resonance condition caused by internal fields arising from various types of anisotropy, with the hope that the results might possibly explain some or all of the anomalous g -values that have been reported.

* See also Griffiths (1951).

The bearing of the present results on the g -value problem may be summarized as follows:

(i) The use of the effective field expressions given in (16) for a conducting disc decreases the experimental discrepancy between g_{\parallel} and g_{\perp} found by Kittel, Yager and Merritt to about 1%. Since this is within the limits of experimental error, g is independent of field orientation. Nevertheless, the value of g found for supermalloy still remains about 2.19.

(ii) The work of Kittel and Herring (1950) and Macdonald (1950) shows that the high g -values found for metals cannot be ascribed to exchange force effects; such an explanation would not, in any event, apply to non-conducting materials.

(iii) The results of experiments on anisotropic single crystals are in excellent agreement with theory. There appears to be no g -dependence upon special crystalline directions, but the experimentally measured g -values are anomalously large.

(iv) The g -factor has been found independent of applied stress, and the use of the correction terms to the effective field found when isotropic plane stress is present in a disc-like ferromagnetic specimen has satisfactorily explained an apparent dependence of g upon evaporated film thickness and upon temperature for nickel discs. Stress effects may also be pertinent to the complete interpretation of the g -value temperature dependence found by Bickford (1950) for magnetite at low temperatures.

(v) The internal field results of the present paper, taken in conjunction with the experimental results of Macdonald on stressed and stress-free nickel films, would seem to render inapplicable Birks' (1948) suggestion that high g -values are due to internal anisotropy or strain fields.

There has been much speculation as to the reason for the anomalously large g -values. A critical survey of the reasons suggested for the effect has been given by Kittel (1949 a), see also Van Vleck (1950). None of the explanations seems able to account for the phenomenon in detail, although it is likely that the effect is connected with spin-orbit coupling as postulated by Polder (1949) and by Kittel (1949 a). There is as yet no detailed theory of the phenomenon, and it remains essentially unexplained.

APPENDIX

EFFECTIVE DEMAGNETIZATION CONSTANTS DUE TO ANISOTROPY

The problem considered here is the transformation of equations (7), (9), (10) and (11) for anisotropy internal-field contributions (referred to the fixed coordinate system (X_1, X_2, X_3)) to the rotated coordinate system (X_1', X_2', X_3') in order to obtain expressions of the form of equation (12) for the internal field contributions in the primed system. The N_{jk}^a demagnetization elements due to anisotropy defined by the transformation may then be used in equation (14) to give the general resonance condition for any orientation between primed and unprimed axes.

(i) *Shape* (cf. equation (7))

Since equation (7) is linear in I_j , the N_j^s shape demagnetization constants may be considered the diagonal elements of the demagnetization matrix referred to the unprimed principal-axis coordinate system. A coordinate transformation of this matrix to the primed system then gives the desired result:

$$N_{jk}^s = \sum_l \gamma_{jl} \gamma_{kl} N_l^s.$$

Therefore the diagonal-element differences appearing in the resonance condition are

$$(N_{11}^s - N_{33}^s) = \sum_l [\gamma_{1l}^2 - \gamma_{3l}^2] N_l^s; \quad (N_{22}^s - N_{33}^s) = \sum_l [\gamma_{2l}^2 - \gamma_{3l}^2] N_l^s. \quad \dots \dots (23)$$

(ii) *Magnetocrystalline Anisotropy: Hexagonal Crystal* (cf. equation (9))

The above procedure for obtaining N_{jk}^a elements is not applicable to this case because \mathbf{H}_c is here not a linear function of the I_j components. Instead, the \mathbf{H}_c vector must be resolved directly in the primed system and the unprimed I_j components occurring in the resulting $(H_c)_j'$ field components expressed in terms of the I_j' components. A linear dependence of the $(H_c)_j'$ components upon I_k' , as in equation (12), is then obtained by dropping terms containing powers of i_k' greater than unity. Such neglect is justified as long as $I_0 \gg i_k'$; this condition holds in practice. One obtains

$$\left. \begin{aligned} (N_{11}^c - N_{33}^c) &= \frac{2}{I_0^2} [(K_1' + 2K_2')(\gamma_{31}^2 - \gamma_{11}^2) + 2K_2'(3\gamma_{11}^2 - \gamma_{31}^2) \gamma_{31}^2], \\ (N_{22}^c - N_{33}^c) &= \frac{2}{I_0^2} [(K_1' + 2K_2')(\gamma_{31}^2 - \gamma_{21}^2) + 2K_2'(3\gamma_{21}^2 - \gamma_{31}^2) \gamma_{31}^2], \end{aligned} \right\} (24)$$

where I has been replaced by I_0 .

(iii) *Magnetocrystalline Anisotropy: Cubic Crystal* (cf. equation (10))

To simplify the succeeding formulae, the following functions of the direction cosines are defined:

$$\begin{aligned} f_{jk} &= \sum_l \gamma_{jl}^2 \gamma_{kl}^2; \quad h_{l,m,n} = \gamma_{l1} \gamma_{m2} \gamma_{n3}, \\ g_{l,m,n} &= h_{l,m,n} [h_{l,m,n} + 2\{h_{m,n,l} + h_{n,l,m}\}]. \end{aligned}$$

The desired demagnetization element differences obtained as in (ii) are then

$$\left. \begin{aligned} (N_{11}^c - N_{33}^c) &= \frac{2}{I_0^2} [K_1(f_{33} - 3f_{13}) - K_2\{g_{3,3,3} - (g_{1,3,3} + g_{3,3,1} + g_{3,1,3})\}], \\ (N_{22}^c - N_{33}^c) &= \frac{2}{I_0^2} [K_1(f_{33} - 3f_{23}) - K_2\{g_{3,3,3} - (g_{2,3,3} + g_{3,3,2} + g_{3,2,3})\}]. \end{aligned} \right\} (25)$$

The terms in K_1 are in agreement with results obtained by Van Vleck (1950) from a quantum-mechanical treatment. General expressions for the second-order terms have not been given previously. As Van Vleck points out, it is noteworthy that the classical and quantum-mechanical treatments give exactly the same results, at least to first order.

(iv) *Strain* (cf. equation (11))

Define the following quantity in terms of the stress tensor elements referred to the unprimed crystal-axis coordinate system:

$$S_{jk} = \frac{3}{I^2} [(\lambda_{100} - \lambda_{111})\delta_{jk} + \lambda_{111}] P_{jk}.$$

Then the expression for the internal field contribution may be written

$$(H_{st})_j = \sum_k S_{jk} I_k.$$

Transforming the matrix $[S_{jk}]$ to the primed system, one obtains

$$N_{jk}^{st} = - \sum_l \sum_m \gamma_{jl} \gamma_{lm} S_{lm}.$$

Therefore the general expressions for the strain demagnetization element differences are

$$(N_{11}^{st} - N_{33}^{st}) = \sum_l \sum_m [\gamma_{3l}\gamma_{3m} - \gamma_{1l}\gamma_{1m}] S_{lm}; \quad (N_{22}^{st} - N_{33}^{st}) = \sum_l \sum_m [\gamma_{3l}\gamma_{3m} - \gamma_{2l}\gamma_{2m}] S_{lm} \dots\dots(26)$$

ACKNOWLEDGMENTS

It is a pleasure to acknowledge the helpful suggestions of Professor M. H. L. Pryce, Drs. J. H. E. Griffiths, C. Kittel and K. W. H. Stevens, and the referees. The author wishes further to thank the Rhodes Trustees for a Rhodes scholarship held during this investigation.

REFERENCES

BECKER, R., and DORING, W., 1939, *Ferromagnetismus* (Berlin : Springer).
 BICKFORD, L. R., Jr., 1950, *Phys. Rev.*, **78**, 449.
 BIRKS, J. B., 1948, *Phys. Rev.*, **74**, 988.
 BLOEMBERGEN, N., 1950, *Phys. Rev.*, **78**, 572.
 BOZORTH, R. M., 1949, *Physica*, **15**, 207.
 BROWN, W. F., 1940, *Phys. Rev.*, **58**, 736.
 GRIFFITHS, J. H. E., 1951, *Physica*, **17**, 253
 HEWITT, W. H., Jr., 1948, *Phys. Rev.*, **73**, 1118.
 KIP, A. F., and ARNOLD, R. D., 1949, *Phys. Rev.*, **75**, 1556.
 KITTEL, C., 1947, *Phys. Rev.*, **71**, 270; 1948, *Ibid.*, **73**, 155; 1949 a, *Ibid.*, **76**, 743; 1949 b, *Rev. Mod. Phys.*, **21**, 541.
 KITTEL, C., and HERRING, C., 1950, *Phys. Rev.*, **77**, 725.
 KITTEL, C., YAGER, W. A., and MERRITT, F. R., 1949, *Physica*, **15**, 256
 LANDAU, L., and LIFSHITZ, E., 1935, *Phys. Z. Sowjet*, **8**, 153
 LIFSHITZ, E., 1944, *Ź. Phys. U.S.S.R.*, **8**, 337.
 LILLEY, B. A., 1950, *Phil. Mag.*, **41**, 792.
 MACDONALD, J. R., 1950, *D.Phil. Thesis* (Oxford); 1951 a, *Phys. Rev.*, **81**, 312; 1951 b, *Ibid.*, **81**, 329.
 OSBORN, J. A., 1945, *Phys. Rev.*, **67**, 351.
 POLDER, D., 1949, *Phil. Mag.*, **40**, 99.
 STONER, E. C., 1945, *Phil. Mag.*, **36**, 803
 STONER, E. C., and WOHLFARTH, E. P., 1948, *Phil. Trans. Roy. Soc. A*, **240**, 599.
 VAN VLECK, J. H., 1950, *Phys. Rev.*, **78**, 266.
 YAGER, W. A., GALT, J. K., MERRITT, F. R., and WOOD, E. A., 1950, *Phys. Rev.*, **80**, 744.